Internal Report I

Evaluation of the Generalised Predictive Controller

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Abstract

At the beginning of the new millennium, the proportional, integral and derivative PID controller remains by far the most dominant form of feedback controller in use today. The control law of PID is very simple but extremely effective and the performance to cost ratio of the technique, when it works, is well appreciated. Plenty of tuning techniques and years of research have resulted in improved understanding and enhanced controller performance, though the basic structure remains the same. Many research contributions purport that advanced controller algorithms e.g. predictive controllers, outperform the conventional PI(D) controller. Invariably, such claims are supported, in simulation, by poorly tuned PI(D) controllers; consequently it is not surprising that the advanced structures yield enhanced performance. This research proposes to compare a well–tuned PID controller with a more advanced structure, namely the generalised predictive controller of Clarke et al. (1987).

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Nomenclature

Δ	Differencing operator $1 - z^{-1}$
λ	Control weighting sequence
ω_B	Bandwidth in frequency domain
ω_c	Frequency where the Nyquist plot intersects the unit circle
ω_n	Natural frequency
ω_s	Sampling frequency
ξ	Damping factor
A	Plant denominator polynomial in backward shift operator z^{-1}
B	Plant numerator polynomial in backward shift operator z^{-1}
d	Time delay of a system (*samples*)
DM	Delay margin
$e(t)$	Control error
$G_c(s)$	Controller transfer function
$G_p(s)$	Process transfer function
G_{gap}	Generation gap in the genetic algorithms
GM	Gain margin
h	Sampling period
i	Imaginary unit
K_d	Derivative gain in PID controller
K_i	Integrating gain in PID controller
K_p	Proportional gain in PID controller
$L(s)$	Transfer function of an open–loop
L_{ind}	Length of the chromosome structure
M_p	Resonance peak of the complementary sensitivity transfer function $T(s)$

M_s	Resonance peak of the sensitivity transfer function $S(s)$
MM	Modulus margin
N_1	Minimum cost horizon for GPC control law
N_2	Maximum cost horizon for GPC control law
N_u	Control horizon
na	Order of the process pole polynomial
nb	Order of the process zero polynomial
P_m	Mutation probability
PM	Phase margin
$S(s)$	Sensitivity transfer function
$T(s)$	Complementary sensitivity transfer function
T_d	Time delay of a system (sec)
T_i	Integration time
T_r	Rise time defined as the time it takes the signal to go from 10% to 90% of the final value
T_s	Settling time at step response
T_{rd}	Settling time at disturbance rejection
$u(t)$	Control signal
$u_d(t)$	Disturbed control signal
$y(t)$	Process output
y_d	Overshoot at disturbance
y_p	Overshoot at step response
y_{max}	Peak amplitude at step response
y_{ss}	Steady state value
CARIMA	Controlled Autoregressive and Integrated Moving Average
erfc	Complementary error function
GA	Genetic Algorithm
GPC	Generalised Predictive Control
IAE	Integrated Absolute Error
ISE	Integrated Squared Error
LQG	Linear Quadratic Gaussian

LRPC	Long Range Predictive Control
MBPC	Model Based Predictive Control
OSAC	One Step Ahead Control
SISO	Single Input Single Output
SUS	Stochastic Universal Sampling

Chapter 1

Introduction

The PI(D) controller is by far the most dominating form of feedback in use today. More than 90% of control loops are of the PID type, the majority of which are PI control [6]. Many tuning technique appeared during last decades and scientists still working on improving this type of control. Many researches for a single loop control use the PID controllers with Ziegler–Nichols tuning method [29] as a benchmark, see [16, 15]. The modern optimization techniques based on the frequency analysis outperform the the Ziegler–Nichols method [3]. An alternative to the PID controller is the Model Based Predictive Controller (MBPC). The idea of MBPC traced to the work of Kalman at al. [11, 12] in early 1960s when Linear Quadratic Gaussian (LQG) concept was introduced. The formulation of the Generalised Predictive Controller (GPC) was proposed by Clarke at al. [8, 9].

Many benchmarks were made in terms of equivalence different tuning techniques with the same controller [21, 16, 27], also comparison the PID controller and the GPC controller had been done [22]. The research examined both controllers in laboratory scale for a single tank system.

Developing computers make possible to use powerful optimization techniques, such as Genetic Algorithms (GA). This technique was successfully applied to tune the PID Controllers [10], MBPC [23] and the GPC controller as well [18].

In 2001 at the IFAC Workshop on Digital Control Karl Åström presented models which are suitable for benchmarks controllers [5]. These models are applied to the benchmark presented at the workshop. The goal of research proposed in this report is a comparison the well known PI controller with the GPC controller at a peer–to–peer level using GA as a tuning method.

This report is organized as follows:

- Chapter 1 Introduction.
- Chapter 2 introduces definition of sampling period and different methods to obtain the correct value, including analysis in time and frequency domain. Next 14 models presented by Karl Åström [3, 5] are described and the proper sampling period for the GPC controller is chosen. The last section of this chapter describes an easy to control first order system, which is not included in Åström's benchmark test. For all models except the model presented in section 2.10 the PI and GPC controller are tuned.

- Chapter 3 presents the typical textbook PI controller and the more advanced structure proposed by Karl Åström [3]. Both are described and the GPC controller is introduced as an alternative to the previous controllers. The control law of the controllers and the tunable parameters are described.

- Chapter 4 describes the tuning of both of these controllers. The performance criteria for servo and regulatory response are introduced. The optimization technique, genetic algorithms, is reported as a tool for tuning both controllers. The basic idea and parts of the code are included. In the last section a short analysis of searching areas, in terms of the GPC controller, for three models is presented.

- Chapter 5 presents the results of tuning both controllers in terms of the performance criteria. The genetic algorithm described in section 4.4 is used as an optimization tool. For the GPC controller three cases of tuning were applied.

Chapter 2

The Åström test – models

The systems which are discussed in this chapter where proposed by Karl Åström [3, 5]. In this internal report models from both publications will be considered – the workshop and the paper as well. The first six models are representative of standard process control systems. All models describe linear processes and are specified by a transfer function $G(s)$ which is analytical with finite poles and possibly an essential singularity at infinity. This description covers finite dimensional systems with time delay (sections 2.4 and 2.9) and infinite dimensional systems described by linear partial differential equations (section 2.10).

2.1 Choice of the sampling period

In terms of the design of a digital controller it is necessary to choose a sampling period. If the sampling period is too long it is more difficult to control the process, especially for the disturbance rejection; it increases the deviation after the load disturbance. Relatively low sampling rates can be used in control problems, because the dynamics of many controlled systems have a low pass character and their time constants are typically larger then the closed–loop response times. However, very short sampling periods increases the load on the computer. The guidelines for the choice of sampling period are presented in following sections.

2.1.1 Settling time

The sampling period, denoted as h, should be less then $1/10^{th}$ of either the open or closed–loop settling time T_s depending on which is shorter. For the oscillatory system presented in section 2.5 this technique is inappropriate, because in general the sampling period will be too long. This is the simplest method and well documented in the literature.

2.1.2 Rise time

The sampling period, h, can be obtained from equation 2.1.

$$h = \frac{T_r}{N_r} \tag{2.1}$$

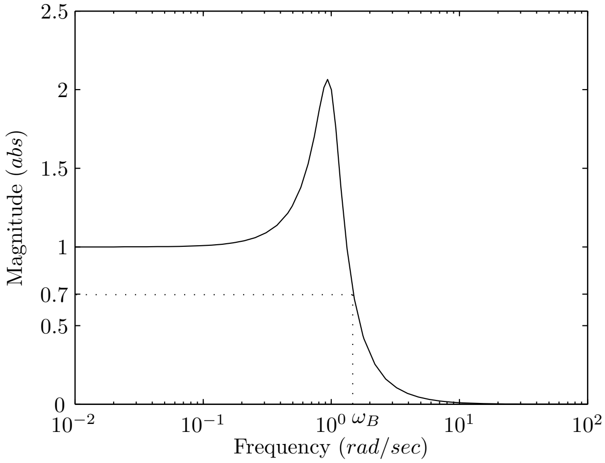


Figure 2.1: Presentation of bandwidth

where T_r is the rise time and N_r is number of sampling periods per rise time. For the first order systems it is reasonable to choose N_r between $4-10$, see [2, sec. 3.7].

2.1.3 Analysis in the frequency domain – the damped frequency method

The sampling period, h, can be related to the damped frequency of the closed–loop system. Given the natural frequency, ω_n, and the damping factor, ξ, the sampling period is achieved from equation 2.2.

$$h = \frac{2 * \pi}{N * \omega_n * \sqrt{1-\xi^2}} \tag{2.2}$$

The parameter N is the ratio of damped period to sampling period. It is reasonable to choose $N = 25 - 75$ [2, sec. 9.2].

2.1.4 Analysis in the frequency domain – the bandwidth method

Another technique based on the frequency domain analysis uses the bandwidth of closed–loop system [2, sec. 2.6]. Illustration of the bandwidth ω_B is shown in figure 2.1 and is defined as the first frequency where the gain drops below 70.79 percent ($-3dB$) of its DC value. Reasonable sampling frequencies are ten to thirty times the bandwidth of the closed–loop system, as it is defined by equation 2.3.

$$10 * \omega_B \leq \omega_s \leq 30 * \omega_B \tag{2.3}$$

where ω_s is the sampling frequency. The sampling period, h, yields

$$h = \frac{2 * \pi}{\omega_s} \tag{2.4}$$

2.1.5 Analysis in the frequency domain – the natural frequency method

An other technique presented in [2, sec. 9.2] states that the sampling period should be chosen as shown in equation 2.5.

$$\omega_n * h = 0.2 - 0.6 \tag{2.5}$$

where ω_n is the desired natural frequency of the closed–loop system. The choice depends on the nature of the disturbance acting on the system.

2.2 System with multiple equal poles

The first of the transfer functions presented by Karl Åström is given by 2.6.

$$G_1(s) = \frac{1}{(s+1)^n}, \quad n = 1, 2, 3, 4, 8 \tag{2.6}$$

They are common in the process industry. For large values of n the system behaves like a system with long time delay. In the paper [2] K. Åström proposed to use $n = 3$, hence the function described by equation 2.7 will be considered.

$$G_1(s) = \frac{1}{(s+1)^3} \tag{2.7}$$

The system is stable in closed–loop when a unit feedback is applied. For the system described by equation 2.7 the rise time is $T_r = 4.22(sec)$, hence the sampling period $h = 0.42 - 1.05(sec)$, based on the rise time analysis given in section 2.1.2. Using the method presented in section 2.1.1 the settling time for the open–loop system is $T_s = 7.52(sec)$, so the sampling period should not be longer than $0.752(sec)$. The frequency analysis method mentioned in section 2.1.3 yields $h = 0.15 - 0.45(sec)$ for closed–loop system presented by equation 2.7, ($\omega_n = 0.638\ rad/sec$, $\xi = 0.5$). The sampling period $h = 0.1(sec)$ was chosen for the model $G_1(s)$.

2.3 Fourth order system

This model is similar to the previous one, but the settling time and the rise time are shorter.

$$G_2(s) = \frac{1}{(s+1)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)} \quad \alpha = 0.1,\ 0.2,\ 0.5,\ 1.0 \tag{2.8}$$

The case where $\alpha = 0.2$, is presented in equation 2.9.

$$G_2(s) = \frac{1}{(s+1)(1+0.2s)(1+0.04s)(1+0.008s)} \tag{2.9}$$

This system has four poles whose separation is determined by the parameter α. For the case where $\alpha = 1$ the system G_2 is identical to the system G_1, for $n = 4$. These two systems, G_1, G_2, represent processes which are relatively easy to control. System G_2 should be sampled at $0.57 - 2.28(sec)$ according to equation 2.1, or $0.15(sec)$ if the settling time method is used. A sampling period of $0.1(sec)$ was chosen.

2.4 Third order system with long dead time

This system is similar to the model G_1, however in this case a time delay, $T_d = 15(sec)$, is included. Since the model dynamics of the systems G_1 and G_3 are the same, the same sampling period, $h = 0.7(sec)$, was chosen. The model is presented in equation 2.10.

$$G_3(s) = \frac{1}{s^3 + 3s^2 + s + 1} * e^{-15s} \tag{2.10}$$

2.5 Integrating system

For the model G_4 it is inappropriate to select the sampling rate based on the settling time criteria presented in section 2.1.1. This is due to the oscillatory behavior of the feedback system with a unit gain. In this case it is better to choose h based on the bandwidth of the closed–loop system. The bandwidth is $1.1(rad/s) = 0.175(Hz)$, so the sampling frequency is $\omega_s = 1.7 - 5.2(Hz)$, yields $h = 0.19 - 0.58(sec)$. The method based on the rise time yields $h = 0.17 - 0.43(sec)$, for the rise time $T_r = 1.74(sec)$ of the closed–loop system with a unit gain. A sampling period of $0.2(sec)$ was chosen for this model.

$$G_4(s) = \frac{1}{s(s+1)^2} \tag{2.11}$$

2.6 Inverse unstable system

The class of non–minimum phase systems is comprised of time delayed systems, unstable systems (poles in the right half of the complex plane) and inverse unstable systems (zeros in the right half of the complex plane).

$$G_5(s) = \frac{1 - \alpha s}{(s+1)^3} \quad \alpha = 0.1,\ 0.2,\ 0.5,\ 1.0,\ 2.0,\ 5.0 \tag{2.12}$$

The systems of equation 2.12 represent an inverse unstable system. These systems have three equal poles and a zero in the right half plane. The achievable performance is determined by the parameter α. The difficulty of control increases with increasing α. [5]. When α is increasing the zero moves to the origin of the pole–zero map. For $\alpha = 0.1$ the closed–loop system $G_5(s)$ has gain margin $15.8dB$, when $\alpha = 2$ the gain margin is equal to $1.16dB$ and when $\alpha = 5$ the system is unstable, the gain margin is $-6dB$. With increasing α the peak response of the transfer function, $G_5(j\omega)$, is increasing as well. In this study the choice $\alpha = 2$ was made, which is presented in equation 2.13.

$$G_5(s) = \frac{1 - 2s}{(s+1)^3} \tag{2.13}$$

For this system the sampling period was chosen as $0.15(sec)$ based on the frequency response analysis.

2.7 An underdamped system

This system is included in [3], however was not presented at the workshop [5].

$$G_6(s) = \frac{9}{(s+1)(s^2+\alpha s+9)} \qquad \alpha = 0.2,\ 0.5,\ 1.0,\ 2.0 \tag{2.14}$$

The system has two complex poles with the relative damping $\xi = \alpha/6$. When the parameter α is decreased the system becomes more difficult to control.

$$G_6(s) = \frac{9}{(s+1)(s^2+2s+9)} \tag{2.15}$$

For the choice $\alpha = 2$, the closed–loop rise time $T_r = 0.62(sec)$ while the open–loop rise time $T_r = 2.18(sec)$. A Frequency response analysis yields the sampling time as $h = 0.06 - 0.19(sec)$, while analysis based on the rise time yields $h = 0.06 - 0.15(sec)$. A sampling period $h = 0.1(sec)$ was chosen in this case.

2.8 Pure time delay

$$G_7(s) = e^{-\alpha s} \quad when \quad \alpha = 1,\ hence \tag{2.16}$$

$$G_7(s) = e^{-s} \tag{2.17}$$

This system and the system presented in section 2.9 were proposed in [3]. These systems are relatively easy to control in closed–loop. For this system, $G_7(s)$, the sampling period was chosen as $0.1(sec)$.

2.9 Pure integrator with time delay

A pure integrator with time delay is another example of a common model. The sampling time was chosen as $0.1(sec)$. The system is integrating in open–loop and stable in a unity feedback configuration.

$$G_8(s) = \frac{e^{-s}}{s} \tag{2.18}$$

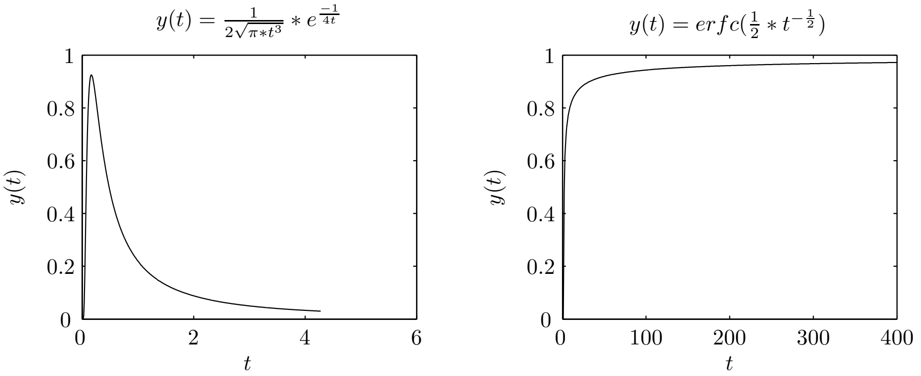
2.10 A distributed parameter system

Transfer functions of the form, equation 2.19, frequently occur when modelling heat conduction problems.

$$G_9(s) = e^{-\sqrt{s}} \tag{2.19}$$

The square root and power functions are not allowed in MATLAB for variables defined as a transfer function. In terms of the system analysis it is necessary to convert this equation to an alternative form. From table [13, sec. 6.10] the transformation is as follows:

$$F(s) = e^{-k\sqrt{s}}(k > 0) \Rightarrow f(t) = \frac{k}{2\sqrt{\pi * t^3}} * e^{\frac{-k^2}{4t}} \tag{2.20}$$



(a) Analysed function

(b) Step response of the analysed function

Figure 2.2: Representation of the analysed function in time domain

$$hence \quad G_9(t) = \frac{1}{2\sqrt{\pi * t^3}} * e^{\frac{-1}{4t}} \tag{2.21}$$

Another way to obtain this function is via the command *ilaplace* from *The Symbolic Math Toolbox version 2.1.3.*

```
ilaplace(exp(-sqrt(s))) ⇒ 1/2/pi^(1/2)/t^(3/2)*exp(-1/4/t)
```

which yields exactly the same result as equation 2.21. Using this toolbox, the step response of the function $G_9(s)$ may be obtained as

```
ilaplace(1/s*exp(-sqrt(s))erfc(1/2/t^(1/2)) ⇒ erfc(1/2/t^(1/2))
```

where *erfc* denotes the *complementary error function.*

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\, dt \tag{2.22}$$

Figure 2.2(b) illustrates the step response using the following command:

```
plot(erfc(1/2./[0:500].^(1/2)))
```

A parametric ARX model was estimated yielding the transfer function 2.23

$$G_9(s) = \frac{0.57s^4 + 13.59s^3 + 5.59s^2 + 0.29s + 0.001}{s^5 + 9.57s^4 + 24.23s^3 + 7.61s^2 + 0.32s + 0.001} \tag{2.23}$$

A 98% fit between the model defined in equation 2.19 and the equation 2.23 was obtained. The step response of the function given by equation 2.23 is illustrated in figure 2.3. This system has not, as of yet, being included in the PI versus GPC comparison.

2.11 Fast and slow modes

The process is described by the following function:

$$G_{10}(s) = \frac{100}{(s+10)^2}\left(\frac{1}{s+1} + \frac{0.5}{s+0.05}\right) \tag{2.24}$$

This system has two fast modes with time constants $0.1(sec)$, one mode with a time constants of $1(sec)$ and a slow mode with a time constant of $20(sec)$. The

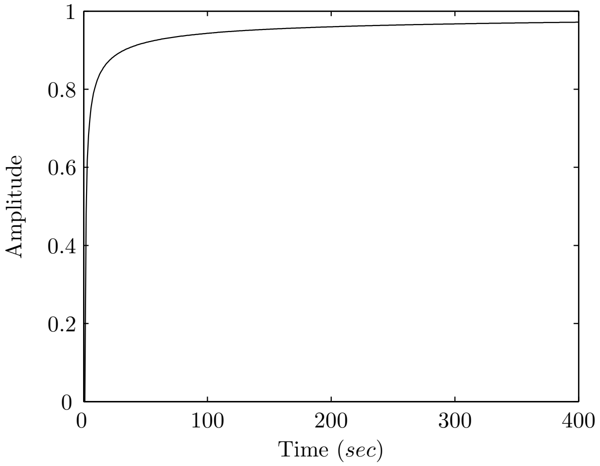


Figure 2.3: Step response of the estimated model $G_9(s)$

system has a static gain of 10 and the step response is dominated by the slow time constants, but it is the faster modes that are critical for the closed–loop system [3]. Thus in this case, the sampling period should be obtained from the closed–loop system. The time constant is $10(sec)$, hence a sampling period of one second was chosen.

2.12 Fast mode

This is a similar model to the previous one, however the slowest term was removed.

$$G_{11}(s) = \frac{150}{(s+10)^2(s+1)} \tag{2.25}$$

The sampling time, $0.1(sec)$, was chosen as one tenth the settling time of the closed–loop system.

2.13 A conditionally stable system

$$G_{12}(s) = \frac{(s+6)^2}{s(s+1)^2(s+36)} \tag{2.26}$$

For this system the frequency response analysis and the rise time method were used to determine a suitable sampling period. The rise time of the closed–loop system is $1.82(sec)$, so dividing this by a factor of $4-10$ gives a sampling time $0.2-0.4(sec)$. From the frequency analysis the sampling period should be $0.17-5.28(sec)$. Finally, the value $0.2(sec)$ was chosen.

2.14 An unstable system

$$G_{13}(s) = \frac{1}{s^2-1} \tag{2.27}$$

Determining a suitable sampling time based on time domain methods fail in this case. The system is unstable in open–loop and unity gain closed–loop as well. It is necessary to resort to the frequency domain analysis and based on this analysis a sampling period of $0.1(sec)$ was chosen.

2.15 First order system

$$G_{14}(s) = \frac{1}{10s + 1} \tag{2.28}$$

A first order transfer function would be representative of liquid level control problems. Closed–loop control of such systems is easy. A sampling time of one second was chosen.

Chapter 3

Controllers

3.1 PI controller

The PI(D) controller is the most popular type of feedback control. Currently more than 90% of control loops are of the PID type [6], the majority of which are PI control. The terms P, I, D, may be interpreted as: P—proportional action which effect the present, I—integral action (past), and D—derivative action (future control error). In the case of pure proportional control, the control action is simply proportional to the control error. The main function of integral action is to make sure that the process output agrees with the set point in steady state. With only proportional control there is normally a control error in steady state. With integral action, a small positive error will always lead to an increasing control signal, and a negative error will give a decreasing control signal no matter how small the error is [4].

For typical control problems such as flow, level etc. this type of controller is sufficient. The typical textbook PI controller has the structure presented in equation 3.1.

$$u(t) = k\Big(e(t) + \frac{1}{T_i}\int\limits_0^t e(t)\,dt\Big) \tag{3.1}$$

where $u(t)$ is the control variable, k and T_i are controller parameters and the control error, $e(t)$, is defined as

$$e(t) = r(t) - y(t) \tag{3.2}$$

where $r(t)$ is the set point and $y(t)$ is the output of a system. To enhance performance and extend the capabilities of the PI controller many alternative formulations have been proposed, e.g. equation 3.3 by Karl Åströ̇m [3].

$$u(t) = k\Big(b * r(t) - y(t)\Big) + k_i\int\limits_0^t \Big(r(t) - y(t)\Big)dt \tag{3.3}$$

where k, k_i, b are controller parameters. The parameter, k_i, is preferred as a tuning knob instead of the integration time, defined as $T_i = k/k_i$ as it looks to reduced computation, however in industry the integration time T_i is more

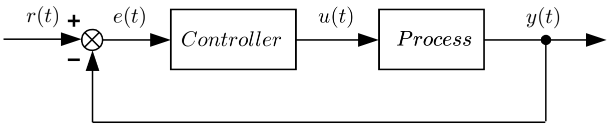


Figure 3.1: Feedback loop

popular. Figure 3.1 illustrates a block diagram representation of the typical feedback loop. The loop consists of a controller, $G_c(s)$, and the model, $G_p(s)$, of a process. The aim of the regulator is to minimize the error which is the difference between the set point $r(t)$ and actual output $y(t)$. In other words, the controller calculates the signal $u(t)$ such that $e(t)$ is zero, despite the occurrence of disturbances and/or changes to the set point command.

3.2 Generalised predictive controller

Model Based Predictive Control, MBPC, is a field which has attracted much research interest and attention since the late 1970's. While different control schemes with predictive capability are now available, one of the more common classes of predictive control is probably the Generalised Predictive Control, GPC, [8, 9]. It has found wide spread applications in areas where it is economically feasible to develop an accurate process model e.g. in the petrochemical industry. These applications are typically described by multivariable, non-linear, constrained process models for which the MBPC philosophy is ideally suited [22], however the GPC is better suited for SISO systems.

3.2.1 Control law

There are three major components in the design of a GPC:

- A model of the system to be controlled. This model is used to predict the system output over the prediction horizon. The GPC uses a CARIMA (Controlled AutoRegressive and Integrated Moving Average) model of the form:

$$A(z^{-1}) * y(t) = B(z^{-1}) * u(t) + C(z^{-1}) * \frac{\xi(t)}{\Delta} \tag{3.4}$$

where

B Numerator of a plant given by a polynomial in the backward shift operator z^{-1}

A Denominator of a plant given by a polynomial in the backward shift operator z^{-1}

$\xi(t)$ uncorrelated random sequence

Δ differencing operator defined as $1 - z^{-1}$. To derive a j–step ahead predictor the Diophantine equation 3.5 is solved.

$$1 = E_j(z^{-1})A\Delta + z^{-j}F_j(z^{-1}) \tag{3.5}$$

where E_j and F_j are the corresponding controller polynomials with degrees $j-1$ and na, respectively.

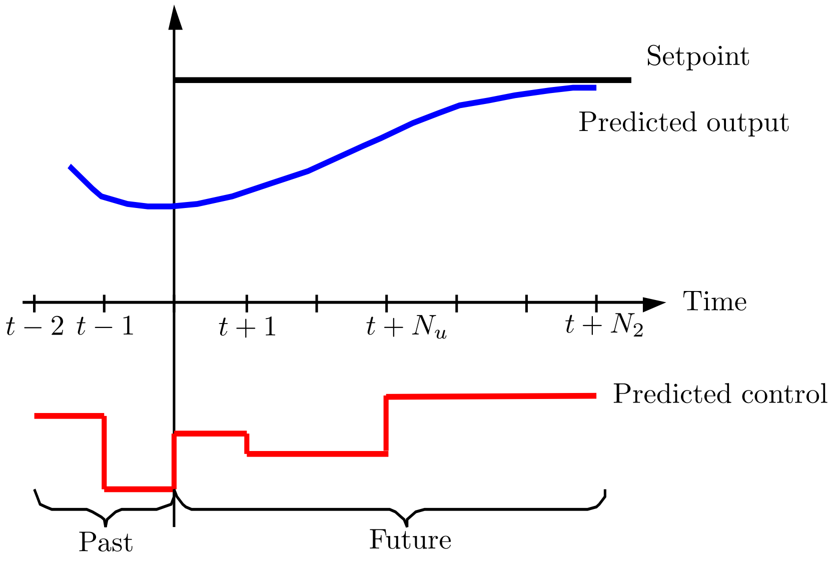


Figure 3.2: Predictive control law

- The various predictive control algorithms have utilized different cost functions. The general aim is that the future output should follow a pre–determined reference signal, taking into account the control effort required to do so. The GPC cost function is given by equation 3.6.

$$J = E\left\{ \sum_{j=N_1}^{N_2} [y(t+1) - w(t+1)]^2 + \sum_{j=1}^{N_u} \lambda(i)[\Delta u(t+j-1)]^2 \right\} \tag{3.6}$$

  where

  N_1 is the minimum cost horizon, subject to $N_1 \leq N_2$

  N_2 is the maximum cost horizon

  N_u is the control horizon, subject to $N_u \leq N_2$

  λ is the control weighting sequence, subject to $\lambda \geq 0$

  w future reference trajectory

- Minimization of criterion function to yield the optimal control output.

This methodology is common to all controllers that belong to the MPC family and is characterized by the strategy represented in figure 3.2.The principle control strategy used is the long range predictive control, LRPC. In this strategy the objective function is expressed in terms of the future values of the output over a certain horizon and of a sequence of future control values. The Generalised Predictive Control, GPC, belongs to this class [14, sec. 7.7] in the sense that the control objective concerns the outputs and the inputs of the plant over a certain horizon in the future beyond $d+1$, where d is the integer delay. These strategies are related to the minimization of a quadratic criterion involving future inputs and outputs in a receding horizon sense. The sequence of control signals in the future is computed, but only the first one is applied and the optimization procedure is restarted at the next step. The generalised predictive control was introduced by Clarke at al. [8, 9].

The control signal, $u(t|t)$, is sent to the process whilst the next control signals calculated are rejected, because at the next sampling instant, $y(t+1)$, is already known and procedure is repeated with this new value and all the sequences are brought up to date. Thus $u(t+1|t+1)$ is calculated (which in principle will be different to $u(t+1|t)$ because of the new information available) using the most recent information. This principle implies that, at each instant t, the prediction horizon N_2 is pushed into the future giving rise to the "receding horizon" concept [19].

3.2.2 GPC tuning knobs

For the classical PI controller structure there are only two tunings parameters, K_p and K_i. Tuning of the GPC controller is more complex, as there are more tuning parameters available. Some of them, like the prediction horizons, only influence the set point response, which other parameters, like the T polynomial, influences disturbance rejection and robustness.

The minimum cost horizon, N_1. In most cases it is recommended to set this parameter to 1, however for systems with dead time, when the delay d is exactly know there is no point setting N_1 to be less than d since there would be unnecessary calculations in that corresponding output cannot be affected by the first action $u(t)$. If d is unknown or is variable, than N_1 can be set to 1 with no loss of stability and the degree of $B(z^{-1})$ increased to encompass all possible values of d [8].

The maximum cost horizon, N_2. If the plant has an initially negative–going nonminimum phase response, N_2 should be chosen so that the later positive–going output samples are included in the cost: in discrete–time this implies that N_2 exceeds the degree of $B(z^{-1})$. In practice, however, a rather larger value of N_2 is suggested, corresponding more closely to the rise time of the plant [8].

The control horizon, N_u. This is an important parameter. For a simple plant (e.g. open–loop stable though with possible dead time and nonminimum phase behavior) a value $N_u = 1$ gives generally acceptable control. In general increasing N_u results in the corresponding output response become more active until a stage is reach where any further increase in N_u makes little difference. An increased value of N_u is more suitable for complex systems where it is found that good control is achieved when N_u is at least equal to the number of unstable or badly damped poles.

The control weighting coefficient, λ. The control weighting coefficient can be used to penalize the control signal. The choice $\lambda = 0$, may result in excessively large control signal moves, which are undesirable.

T **polynomial** is principally used to negotiate the trade–of between disturbance rejection and robustness. It does not effect the servo performance of the GPC. The observer should not be selected on the basis of good robustness properties alone, because of its influence on the speed of disturbance rejection; slow observer poles are to be preferred for robustness, but fast poles for good disturbance rejection. Thus a trade–of between the two must exist when designing an appropriate T polynomial [26].

Chapter 4

Controller tuning

4.1 Performance analysis

In this section definitions of performance criteria at servo and regulatory response are presented. The servo response refers to the closed–loop response, $y(t)$, to an applied set point input $r(t)$, see figure 4.2. The following criteria are considered:

Rise time, T_r, is either defined as the inverse of the largest slope of the step response or the time it takes the response to pass from 10% to 90% of its steady state value y_{ss}.

Settling time, T_s, is the time it takes for the step response to reach and remain within 2% of its steady state value.

Overshoot, y_p, is the ratio of the difference between the first peak and the steady state value, y_{ss}, of the step response.

$$y_p = \frac{y_{max} - y_{ss}}{y_{ss}} \tag{4.1}$$

In industrial control applications it is common to specify an overshoot less than 10%. In many situations it is desirable, however, to have an overdamped response with no overshoot [4].

Integrated Error, IE, is defined by equation 4.2

$$IE = \int_0^\infty e(t)\ dt \tag{4.2}$$

where $e(t)$ is defined by equation 3.2. The IE criterion is a natural choice for control of quality variables for a process where the product is sent to a mixing tank. The criterion may be strongly misleading, however, in other situations. It will be zero for an oscillatory system with no damping.

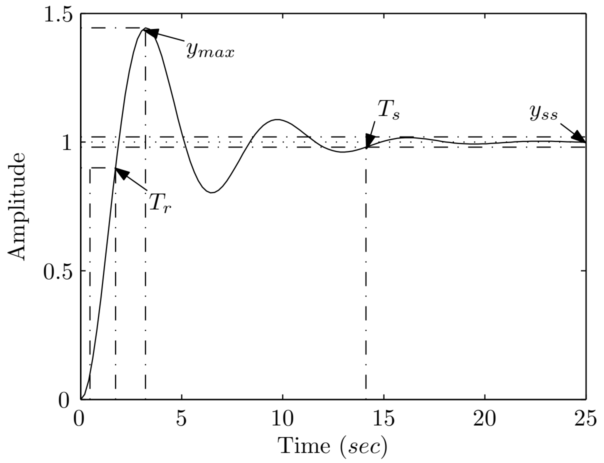


Figure 4.1: Typical response $y(t)$ to the step input $r(t)$

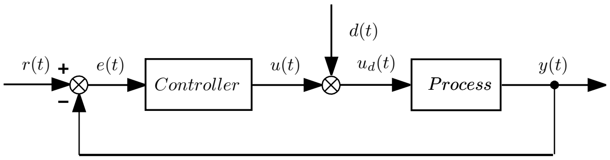


Figure 4.2: The block diagram of regulatory response

Integrated Absolute Error, IAE, this is another very common performance criterion, defined by equation 4.3.

$$IAE = \int_0^\infty |e(t)|\, dt \tag{4.3}$$

Regulatory response. Performance criteria for the regulatory response will now be presented, where the regulatory response is defined as the closed–loop response $y(t)$ to a disturbance input, $d(t)$, as illustrated in figure 4.2 where $d(t)$ is the step disturbance applied at the beginning of simulation, $u(t)$ is the control signal and $u_d(t)$ is the disturbed control signal and $y(t)$ is the output. In this case the following criteria are considered:

Overshoot at disturbance, y_d, is defined as $y_d = -\frac{e_2}{e_1}$, where the peaks e_1 and e_2 are presented in figure 4.3. This definition is taken from [1].

Disturbance settling time, T_{rd}, is the time it takes for the disturbance response to reach and remain within 2% of its steady state value (zero) [1].

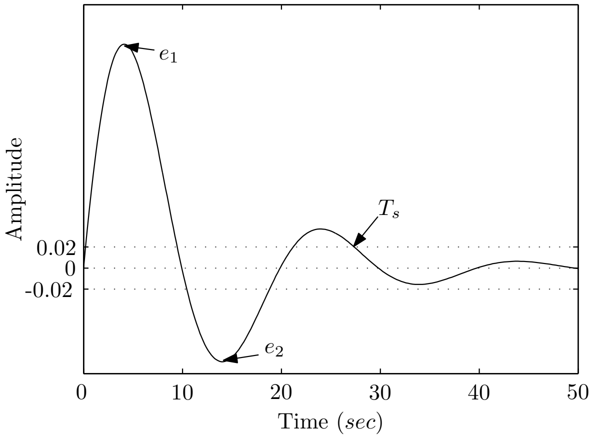


Figure 4.3: Typical response of control error to step or impulse disturbance

4.2 Robust analysis

In practice models are not perfect, a discrepancy will always exist between the identified model and actual plant. For the controller, which is designed using the identified model, to be successful on the actual plant, the controller must be robust to the process missmodeling. Standard measures of robustness are presented in the following sections.

Gain Margin, GM, is defined as the factor by which the gain must change to force the system to marginally stability [24, 4], the GM is defined by equation 4.4

$$GM = \frac{1}{\mid L(i\omega_u) \mid} \tag{4.4}$$

where the ultimate frequency, ω_u, is the frequency where $L(i\omega_u) = -\pi$ and L is defined as an open–loop transfer function described by equation 4.5.

$$L(s) = G_c(s) * G_p(s) \tag{4.5}$$

The gain margin can increase or decrease to maintain stability. The case when the gain has to decrease in dB to loose stability is called the upward gain margin. Example where the gain must increase (absolute value of the gain has to be more than zero and less than one) to lose stability, is called the gain reduction margin or downward margin [15]. To illustrate this consider the following non–minimum phase system:

$$L(z^{-1}) = \frac{0.3758z^{-1} + 0.5204z^{-2} - 0.6413z^{-3} + 0.1497z^{-4}}{1 - 0.2851z^{-1} - 1.055z^{-2} + 0.7687z^{-3} - 0.1475z^{-4}} \quad h = 0.7(sec)$$

The Nyquist plot of this function is presented in figure 4.4. For this example the upward gain margin is equal to $6.6(dB)$ (2.138 of absolute value) and downward margin $-2.69(dB)$ (0.7337). Increasing the gain by a factor 2.138 or decreasing by 0.7337 makes the system conditionally stable. Function *margin* from the *Control System Toolbox* always returns the nearest gain to the critical point $(-1,\ i0)$ on the Nyquist diagram. Therefore, it is possible that gain margin expressed in dB is negative and the system is still stable.

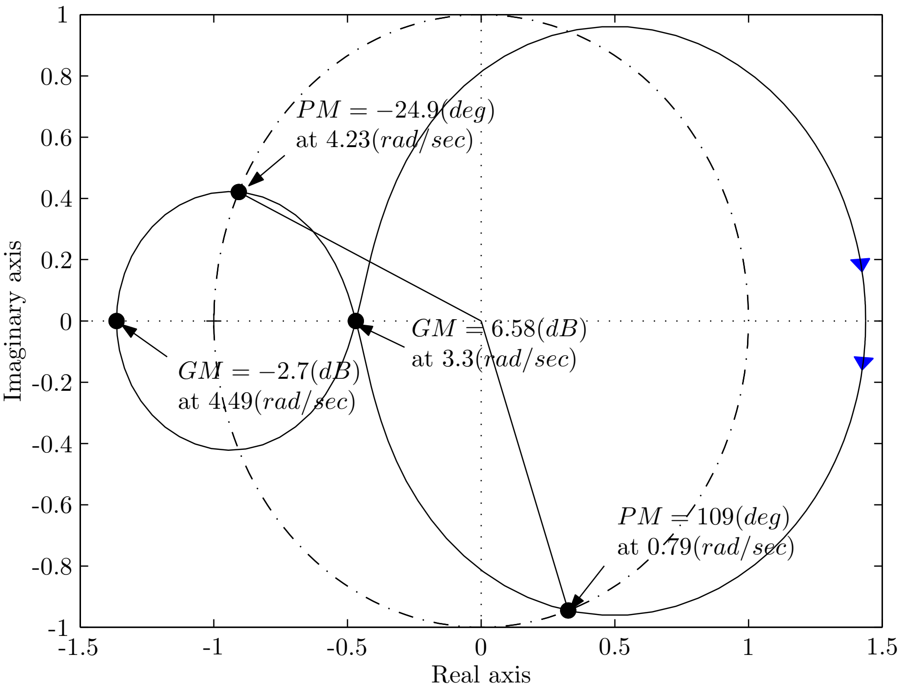


Figure 4.4: Nyquist diagram, illustration of upward and downward gain margin

Phase margin, PM**,** is defined as the angle through which the Nyquist diagram must be rotated such that the diagram intersects the $(-1,\ i0)$ point [24, 4].

$$PM = \pi + \arg\left(L(i\omega_c)\right) \tag{4.6}$$

The gain cross–over frequency ω_c is the frequency where the Nyquist plot intersects the unit circle $|L(i\omega)| = 1$. Typical values of PM range from 30^0 to 60^0, however as presented on figure 4.4 the phase margin could be below zero as well and the feedback system with unit gain remain stable. The gain and phase margin definitions for discrete time systems and for continuous time systems are exactly the same.

Delay margin, DM**,** represents the additional time delay that the closed–loop system can tolerate prior to instability. In this report the delay margin is presented in seconds. The delay margin can be obtained from a frequency domain analysis, i.e. for any frequency, the phase lag introduced by a pure time delay τ is:

$$\angle\phi(\omega) = \omega\tau \tag{4.7}$$

Therefore to convert the phase margin to a delay margin i.e. to compute the additional delay which will lead to instability, equation 4.8 is used.

$$DM = \left|\frac{PM}{\omega_c}\right| \tag{4.8}$$

If the Nyquist plot intercepts the unit circle at several frequencies, ω_c^i characterized by corresponding phase margins of PM_i, the delay margin is defined by:

$$DM = \min_i \left| \frac{PM_i}{\omega_c^i} \right| \tag{4.9}$$

The delay margin should be greater than one sampling period for discrete time systems [14, sec. 8.3.1].

Modulus margin, MM, is defined as the radius of the circle centred on the critical point $(-1,\ i0)$ and tangent to the Nyquist plot of the open–loop transfer function $L(s)$. Recommended practical values for the modulus margin are $MM \geq 0.5(-6dB)$ $[min:\ 0.4(-8dB)]$, see [14]. The inverse of the MM is M_s. This is the maximum value on the Bode plot of the sensitivity function $S(s)$.

$$M_s = \max_{0 \leq \omega \leq \infty} \Big(S(i\omega) \Big) \tag{4.10}$$

where

$$S(s) = \frac{1}{1 + G_c(s) * G_p(s)} \tag{4.11}$$

Reasonable values of M_s are in the range from 1.3 (robust tuning) to 2.0 (more aggressive tuning) [4].

M_p is defined as the resonance peak of the closed–loop system. The value of M_p is the size of the resonance peak of the closed–loop system obtained from frequency domain analysis of the complementary sensitivity function $T(s)$ detailed in equation 4.13.

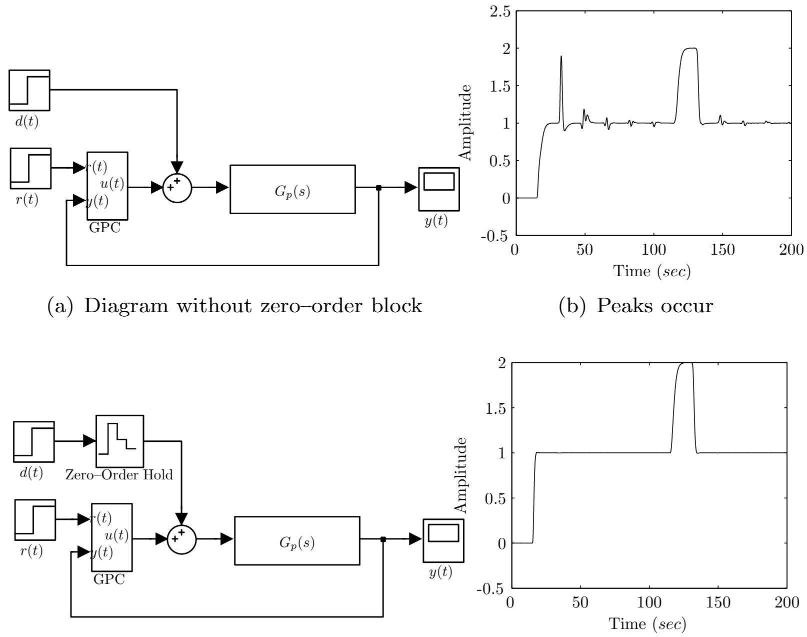
$$M_p = \max_{0 \leq \omega \leq \infty} \Big(T(i\omega) \Big) \tag{4.12}$$

where

$$T(s) = \frac{G_c(s) * G_p(s)}{1 + G_c(s) * G_p(s)} \tag{4.13}$$

4.3 Tuning systems with time delay

The block diagram of figure 4.5(a) was used during the initial controller design stage. In this figure the input $d(t)$ represents a step disturbance, applied at time $t = 100(sec)$. A simulation of this closed–loop system revealed peaks in the closed–loop system response which occurred every 16 seconds, figure 4.5(b), These peaks occurred even if the disturbance, $d(t)$ was set to zero, however if the summation block was removed, the peaks disappeared. A remedy was sought and found, the solution involved including a zero–order hold block between the summation block and the disturbance input, figure 4.5(c). The resulting closed–loop system response is illustrated in figure 4.5(d).



(a) Diagram without zero–order block

(b) Peaks occur

(c) Diagram with zero–order block

(d) Peaks do not occur

Figure 4.5: Peaks in systems with long time delays

4.4 Genetic algorithms

Different problems can be divided into two groups, first solvable and second unsolvable. In reality, what is solvable in theory, need not to be solvable in a sensible time frame (the time of solve can be equal to infinity). Sometimes this complexity of calculations is very useful, because it may be used for security coding.

The Genetic Algorithms (GA) are stochastic global search methods based on the mechanics of natural selection and natural genetics. They are iterative methods, widely used in optimization problems in several branches of the sciences and technologies. Contrary to other methods, in this methodology, at each iteration (generation) a set of solutions defining a population or populations of individuals, is considered. These individuals are ranked, according to the quality of the solution that each one leads to [17]. The *Genetic Algorithm Toolbox* which was used, was designed by Andrew Chipperfield et al. from University of Sheffield [7]. A flow chart of the GA is shown in flow chart 4.6.

1. **Generate initial population.** The GA starts with the generation of an initial population, which describes the possible set of solutions by the binary chromosomes. A fixed number of GA parameters have to be assigned at this stage of the code. For the results presented in this report the following constants were assumed:

   - *Size of population* If the population size is small and the problem is complex (non–convex) the GA has less genetic material to work

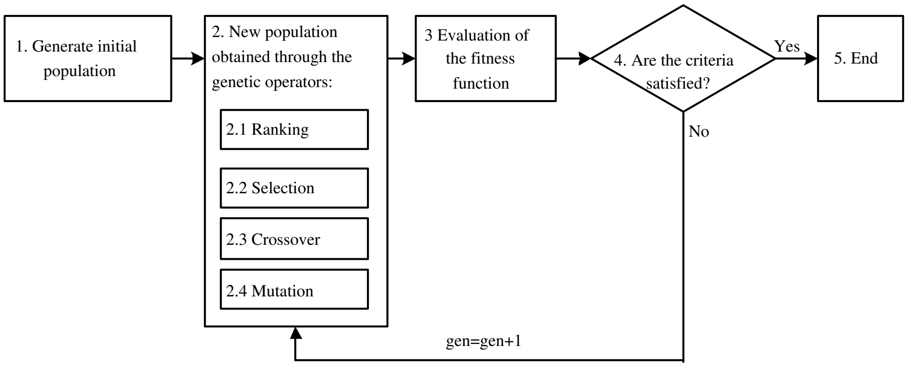

1. Generate initial population
2. New population obtained through the genetic operators:
2.1 Ranking
2.2 Selection
2.3 Crossover
2.4 Mutation
3 Evaluation of the fitness function
4. Are the criteria satisfied?
Yes
5. End
No
gen=gen+1

Figure 4.6: Flow chart of the genetic algorithm

with. Typically, a population is composed of between 30 and 100 individuals [7].

- *Number of variables* was changed depending on the designed controller. For the PI controller only two variables were tuned. For the more complex regulator, such as the GPC controller, eight tuning variables were optimized.
- *Number of individuals replaced by the new generation.* To guarantee convergence it is common to transfer a number of individuals to the next generation unaltered. In the application considered here the five best individuals are automatically passed to the next generation.
- *Resolution of tuning variables.* For T polynomial and the tuning variables of the PI controller the resolution was set to 0.001, while for λ it was set to 0.1.
- *Coding.* The binary representations of chromosomes was used, however the gray code is also available in the GA toolbox.

2.1 Ranking Rank individuals according to their objective fitness values. The best individual is ranked as 2, and the worse as 0. Linear ranking gives good results, however nonlinear ranking is also available.

2.2 Selection The individuals chosen for breeding are selected based on their fitness or ranking. Fitter individuals are assigned a higher probability and are more likely to be selected, however less fit individuals may also be chosen. This ensures genetic variety. A common selection strategy known as *Stochastic Universal Sampling, SUS,* was used. To make sure that the best individuals go to the next generation a generation gap G_{gap} is set and defined in equation 4.14.

$$G_{gap} = \left(\frac{Population\, Size - Number\, of\, the\, best\, individuals}{Population\, Size} \right) \tag{4.14}$$

where the number of the best individuals e.g. 1 to 5 are passed from the current generation to the next one unchanged.

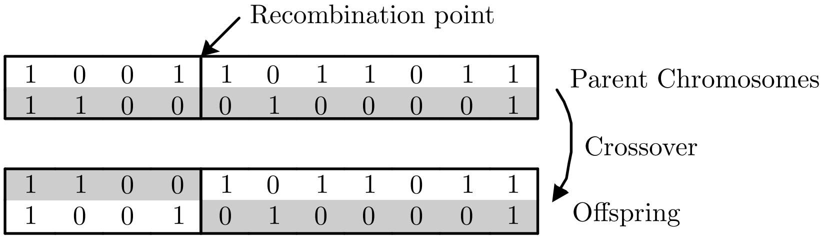


Figure 4.7: Example of single point crossover

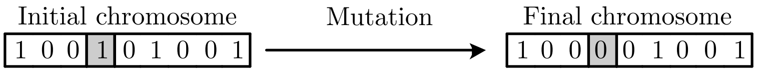


Figure 4.8: Example of mutation

2.3 Crossover The crossover operator recombines pairs of individuals with given probability to produce the offspring. The crossover rate was set to 0.7, as the default value according to the GA toolbox [7]. The simplest recombination is to employ a single point crossover, as illustrated in figure 4.7.

2.4 Mutation The mutation introduces a new genetic material by random changes. THis operator enables a better exploration of the search space.It works on each individual by alternating the value of a randomly selected bit position, see figure 4.8. The mutation probability, P_m, was obtained from equation 4.15.

$$P_m = \frac{0.7}{L_{ind}} \tag{4.15}$$

where L_{ind} is the length of the chromosome structure. Typically the probability for bit mutation is in range 0.001 to 0.01, see [20].

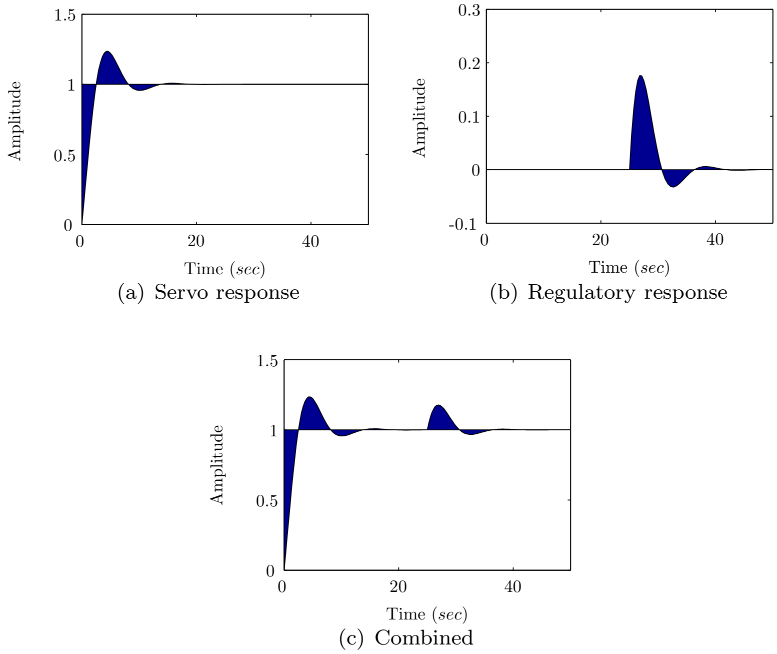
3 Evaluation of the fitness function The *fitness function* indicates how good a candidate solution is. This function, described in section 4.4.1, in classical optimization technique is called an objective function.

Check conditions The genetic algorithm stops when some criterion are reached. This step is described in section 4.4.4.

4.4.1 Cost function

The success of the genetic algorithm as a tuning method is critically depended on the objective function (cost function). There are an infinitive number of criteria, however a few are common; one of them being the IAE described by equation 4.3.

The IAE criterion is in many cases a natural choice, at least for control of quality variables. A severe drawback is that its evaluation requires significant



(a) Servo response

(b) Regulatory response

(c) Combined

Figure 4.9: Presentation of the Integrated Absolute Error

computations or simulation time. The IAE can be applied to one of the following three scenarios :

- IAE at servo response, see figure 4.9(a)
- IAE at regulatory response, figure 4.9(b)
- Combined, figure 4.9(c)

For these three cases a PI and GPC controller were tuned via the GA.

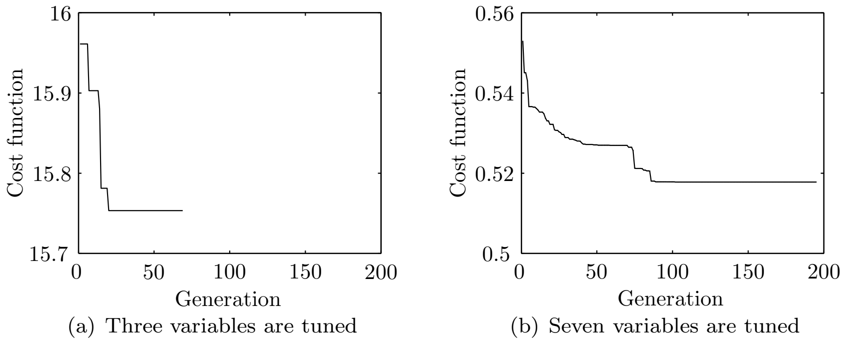
4.4.2 Cancellation of the output horizon

The genetic algorithm does not have information about the problem which it is going to solve. The results are based on continuous evaluations of the objective function. It is necessary to assume that the output predictive horizon has to be more or equal to the initial predictive horizon $N_1 \leq N_2$. The GA works on a probabilistic method, hence there is possibility that $N_1 \leq N_2$, what is forbidden by the GPC control law. The following statement was implemented to the GA:

```
if N2<N1
    N2=N1+N2-1
end
```

4.4.3 Number of bits

In the GA the resolution for each parameter is described by the number of bits and the upper and lower bounds on that parameter. Hence, once the upper and



(a) Three variables are tuned

(b) Seven variables are tuned

Figure 4.10: Error the best individuals

lower bounds have been decided the number of bits must be carefully chosen to ensure a resolution of unity. The number of bits is defined as:

$$N_{bits} = \log_2 \left(\frac{N_{up} - N_{down}}{R} + 1 \right) \tag{4.16}$$

where N_{up} defines the upper bound on the search area, N_{down} the lower bound, and R is the resolution.

4.4.4 Terminating the genetic algorithm

The GA runs in the loop as long as the predefined user criteria are not achieved, see flow chart 4.6. The algorithm may be terminated if:

- a predefined maximum number of the generations has been reached,
- the algorithm has converged e.g. the cost function over a specified number of generations has remaind unchanged,
- GA has reached a prespecified minimum value of the cost function.

A combination of the first two criteria was used e.g.

```
if gen>50
    if Best(gen-49)==Best(gen)
        gen=MaxGen;
    end
end
```

where $MaxGen$ was predefined as $MaxGen = 200$. As shown if figure 4.10(a) if the algorithm converged quickly the GA terminated prior to the predesigned value. In this case the GA terminated at the 69^{th} generation. In most of the analysed cases the maximum number of generations was not achieved. When the T polynomial was included for tuning, the problem became more complex, hence the number of required iterations was larger. This is shown in figure 4.10(b).

	$G_1(s)$	
	$N_u = 1$	$N_u = 3$
IAE	$1.694_{\{N_1=1,N_2=2\}}$	$1.381_{\{N_1=2,N_2=4\}}$
MM	$0.252_{\{N_1=1,N_2=2\}}$	$0.132_{\{N_1=2,N_2=4\}}$

	$G_5(s)$	
	$N_u = 1$	$N_u = 3$
IAE	$6.8828_{\{N_1=18,N_2=19\}}$	$5.711_{\{N_1=6,N_2=20\}}$
MM	$5.3 * 10^{-3}_{\{N_1=18,N_2=19\}}$	$5.51 * 10^{-3}_{\{N_1=6,N_2=20\}}$

	$G_{10}(s)$	
	$N_u = 1$	$N_u = 2$
IAE	$3.2217_{\{N_1=1,N_2=2\}}$	$2.0417_{\{N_1=1,N_2=2\}}$
MM	$0.2454_{\{N_1=1,N_2=2\}}$	$0.2925_{\{N_1=1,N_2=2\}}$

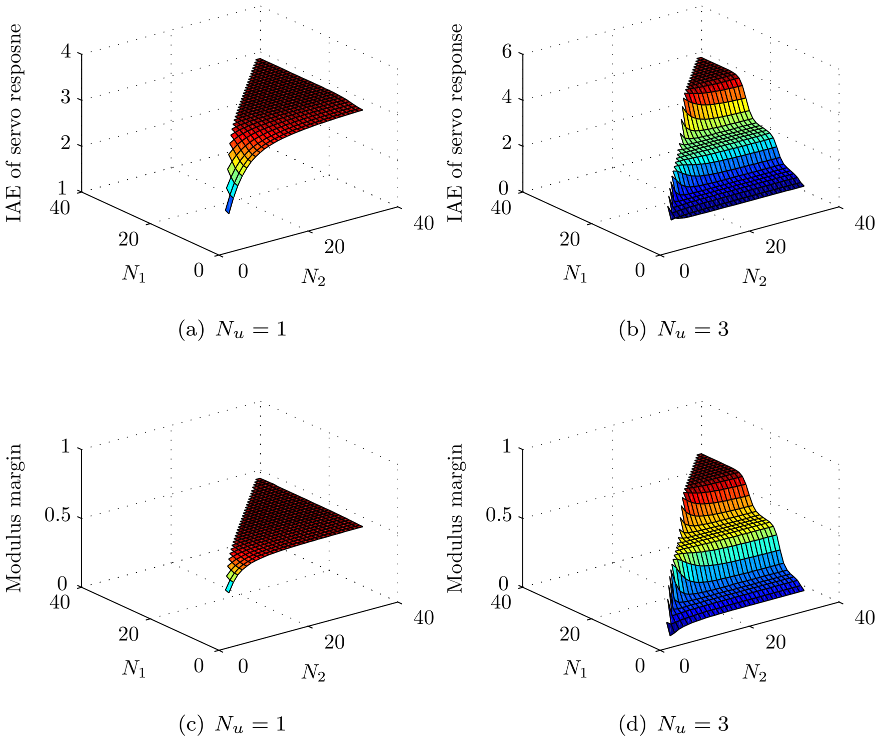
Table 4.1: Minimum values integrated absolute error and modulus margin for selected systems

4.5 Tuning of the GPC controller

Numerous methods are available for tuning the GPC controller. Seven of them were studied by Rani et al. [25], whilst other techniques, such as the genetic algorithm, also exist. The variables like N_1, N_2, λ and the third order T polynomial were tuned. To demonstrate how the prediction and control horizon effect the searching area, the transfer functions $G_1(s)$, $G_5(s)$ and $G_{10}(s)$ are considered. The results are presented in table 4.1.

Tuning model $G_1(s)$ The solution space, as a function of parameters N_1, N_2 is illustrated in figure 4.11. The first row, figures 4.11(a) and 4.11(b), illustrates the IAE of the servo response as a function of $\{N_1,\ N_2\}$ while the second row shows how variations in $\{N_1,\ N_2\}$ influence the modulus margin. The minimum value of the matrix presented in figure 4.11(a) is for $N_1 = 1$ and $N_2 = 2$, the control horizon in this case is set to 1 and the T polynomial does not affect the servo response, hence it is set to 1. For small values of N_1 and N_2 the IAE is small, which means that the servo response is very fast, but the modulus margin, presented in figure 4.11(c), decreases so the system is less stable. For the minimum IAE, the modulus margin was also minimised, which is undesirable.

The second column, figures 4.11(b) and 4.11(d), presents the case where the control horizon N_u is set to three. In figure 4.11(b) the minimum IAE is for $N_1 = 2$ and $N_2 = 4$ and the modulus margin for the same parameters is smaller, so the system is less stable. Thus incorporating the control horizon is beneficial in terms of optimizing the servo performance, however the standard performance versus robustness trade off remains. Similar results were obtained for systems $G_2(s)$ and $G_3(s)$.



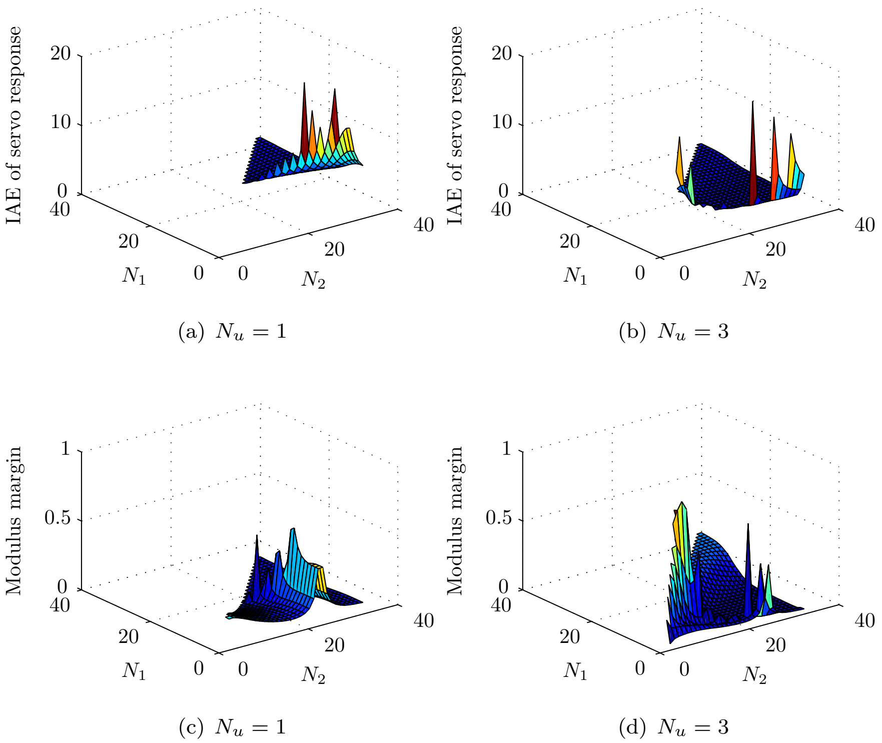
(a) $N_u = 1$

(b) $N_u = 3$

(c) $N_u = 1$

(d) $N_u = 3$

Figure 4.11: Study of a searching area for the system G_1



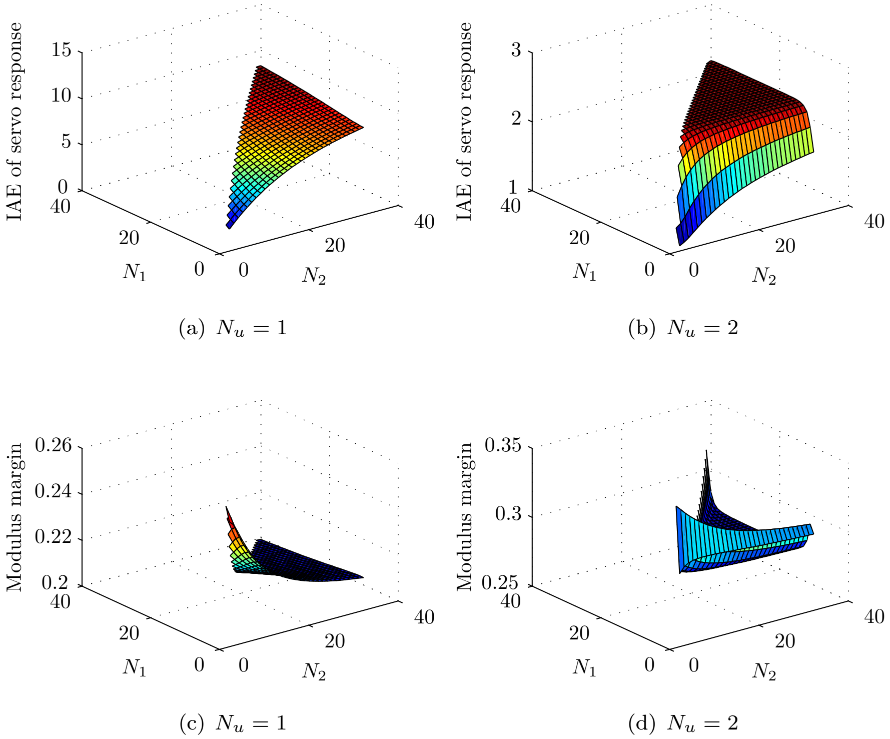
(a) $N_u = 1$

(b) $N_u = 3$

(c) $N_u = 1$

(d) $N_u = 3$

Figure 4.12: Study of a searching area for the system G_5



(a) $N_u = 1$

(b) $N_u = 2$

(c) $N_u = 1$

(d) $N_u = 2$

Figure 4.13: Study of a searching space for the system G_{10}

Tuning model $G_5(s)$ An example of non–convex surface is presented by the inverse unstable system $G_5(s)$ defined by equation 2.13. Figures 4.12(a) and 4.12(b) present the IAE of the servo response for the system $G_5(s)$. In this case smaller values of $\{N_1,\ N_2\}$ does not necessarily yield enhance servo performance. In figure 4.12(a) when the control horizon is set to 1 the minimum IAE is for $N_1 = 18$ $N_2 = 19$. When the control horizon is set to three the searching area looks different, the minimum IAE is for $N_1 = 6$ $N_2 = 20$. The search area of this system is very complex, hence this is difficult to find a minimum for this system. Tuning of this system presents a good test for applying the genetic algorithm. For the systems examined in this report it was found that, in general, as N_2 decreased, the modulus margin decreased as well, which is undesirable. The exception is $G_{10}(s)$, which is presented in the next paragraph.

Tuning model $G_{10}(s)$ The IAE and the modulus margin do not always decrease simultaneously. The effect of $\{N_1, N_2\}$ on the IAE and the modulus margin is presented in figure 4.13. Examining 4.13(a) and 4.13(c) it is evident that as the minimum IAE and maximum value of MM coincide for $N_1 = 1$, $N_2 = 2$. The minimum IAE occurs for $N_1 = 1$, $N_2 = 2$ when control horizon is $N_u = 1$. The surface plot area alternated dramatically by this small change in N_u. If N_u is increased to $N_u = 2$ the surface plot is altered dramatically, however the minimum IAE still occurs for $N_1 = 1$, $N_2 = 2$ and the MM is also quite large for these parameters.

For models $G_1(s)$ and G_{10} common optimisation techniques such as quadratic and linear programming can be used. For models such as $G_5(s)$, where the optimization surface is non–convex the classical methods will not always find a global solution. This statement also applies if the dimension of the search space increases e.g. if λ or the T polynomial are included.

Chapter 5

Results

Firstly, the structure and theoretical bases of both controllers will be compared and the performance comparison will be presented in the next section. Tables with results for the PI controller are in appendix B.1 and for the GPC controller B.2, B.3 and B.4.

When comparing two controllers it is imperative that they are compared at a peer-to-peer level. For this reason the popular PI tuning rules such as those proposed by Ziegler and Nichols are abandoned for the much more powerful optimisation technique such as the Genetic Algorithms (GA). Both controllers have been tuned using the same algorithm. In other words properties of the GA like the population size, the generation gap, maximum number of generations etc., are the same for both tuning techniques. The same objective function was used in both cases evaluate like with like. This enables a fair comparison of both control laws [22].

The PID control law was presented in section 3.1 and details of GPC control law are given in section 3.2. Here only the main properties of both of them will be considered. Both controllers use a model of the process. The model has to be obtained to tune both controllers. The PI controller does not have information about the future behavior of the plant, in comparison with the GPC controller which has that information. For the GPC controller a number of tuning knobs is available, contrasting to the PI case where only K_p and K_i are tuneable. The structure and control law of the GPC controller is much more complex than the PI controller.

5.1 PI controller in comparison with the GPC controller

Figure 5.3 presents the IAE of servo and regulatory response as well as the modulus margin. The white bars illustrate results for the PI controller and the black for the GPC. Models $G_3(s)$ and $G_{13}(s)$ are not compared in figures 5.3(a) and 5.1(a) due to readability the bar chart. The GPC controller outperforms the PI controller in all analysed cases exclude the first order system, $G_{14}(s)$, when only IAE is considered. In terms of robustness, where the MM denotes how robust the controller is, the PI controller surpasses the GPC controller in most cases. Modulus margin results, figure 5.3(b), for models $G_7(s)$ and $G_{10}(s)$

are quite similar. Generally the GPC controller has faster response, however is less robust. Note that the cost function of the GA was described only as the IAE and robust criteria were not included, see section 4.4.1. The case when the cost function is described as regulatory response is analysed in figure 5.2(a). Similarly to figure 5.3(a) the GPC controller for analysed systems outperforms the PI controller and is less robust.

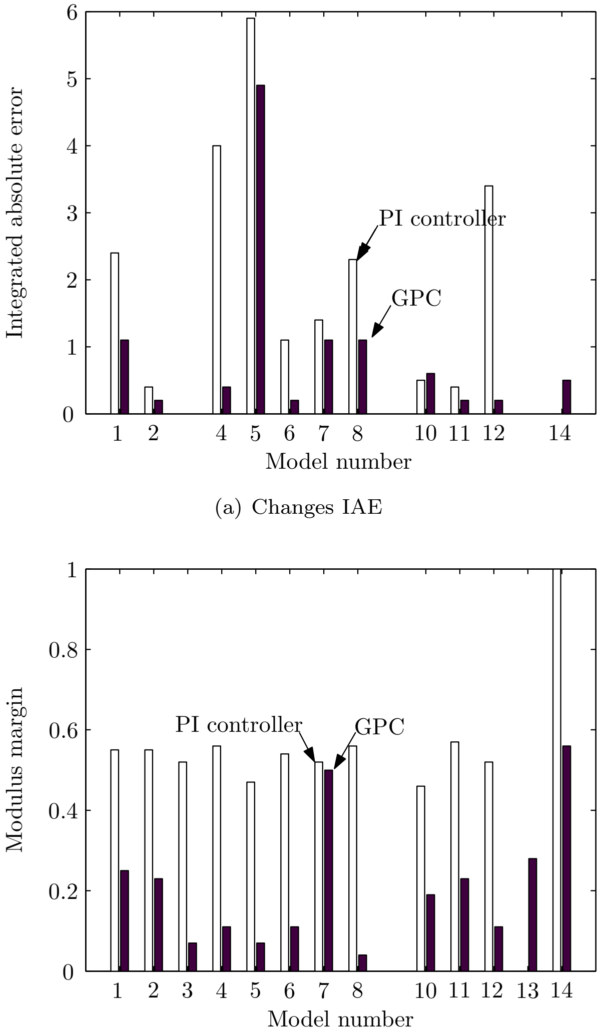
The results are compared based upon optimizing the servo response, then the GPC controller outperforms the PI controller in 12 of the 13 cases studied, the exception being $G_{14}(s)$. When the control horizon is included as a variable, the IAE optimized values are better in comparison to the tuning where the N_u is fixed.

There is no additional improvement of the GPC controller when the P numerator is included and the cost function is described as equation 4.3. The tuning models with P numerator were only for combined case (servo response and load disturbance).

An example of the OSAC controller is for models $G_4(s)$, $G_6(s)$, $G_{14}(s)$ and $G_{10}(s)$ when the cost function was described as IAE servo response and sum of both responses (servo and regulatory), see the table in appendix B.2. When the control horizon, N_u, was included as a tuning knob only designed controller for first order system, $G_{14}(s)$, behaved like the OSAC, appendix B.3. When the T polynomial and the P numerator were tuned the OSAC strategy did not appear in any of the analysed models, appendix B.4.

Tuning of model $G_1(s)$ defined by equation 2.7 was presented for instance by Ya–Gang Wang and et al. [28]. In this paper the design of the PI controller is based on optimization of load disturbance rejection with constrains that the Nyquist curve of the loop transfer function is tangent to a line parallel to the imagine axes in the left - half of the complex plane.

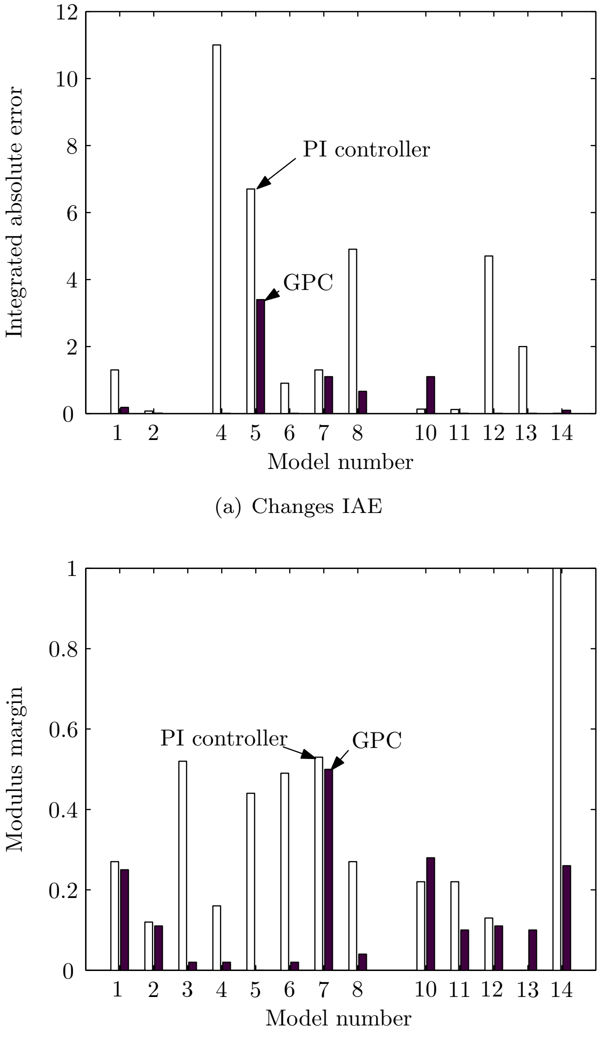
The conclusion is that for more complex systems the servo response and disturbance rejection of the GPC controller is better than the PI controller. When the robust parameters are considered, only the system with pure time delay, G_7, satisfies the recommendations ($MM > 0.5$, $GM > 6dB$, etc.). For model with time delay, G_3, the initial control horizon, N_1, was constrained from 22 samples.



(a) Changes IAE

(b) Changes modulus margin

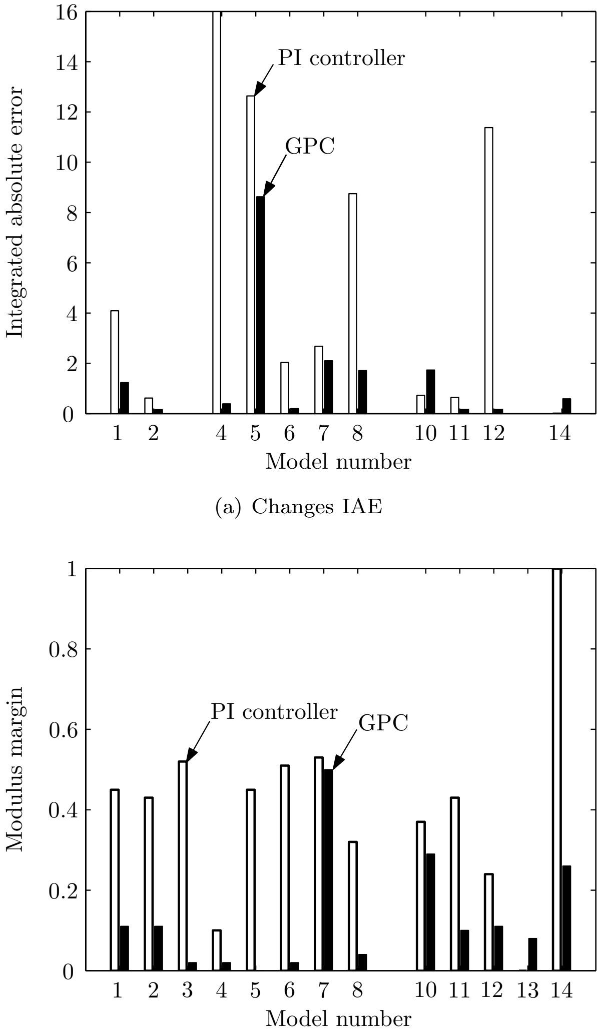
Figure 5.1: Bar chart when cost function is described as IAE servo response



(a) Changes IAE

(b) Changes modulus margin

Figure 5.2: Bar chart when cost function is described as IAE regulatory disturbance



(a) Changes IAE

(b) Changes modulus margin

Figure 5.3: Bar chart when cost function is described as IAE servo response plus IAE regulatory disturbance

Chapter 6

Conclusions

The benchmark was successfully applied to the models. The GPC controller has faster response than the PI controller of the models analysed except the first order system. The benchmark also introduced the author to the robust control problem. Good servo response and disturbance rejection does not indicate that the robust parameters are satisfied. In real time systems, the optimised tuning parameters for the GPC controller may not be as good as during simulations. The research gave an idea to tune controllers not only depend on servo response or disturbance rejection, but also include important robust parameters to the cost function and trade off them.

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Appendix A

An example of GPC implementation

In this appendix is shown an implementation of GPC algorithm to linear control law via a numerical example. It is assumed that the GPC tuning knobs are set as follows:

- Minimum cost horizon, N_1 is set to 1
- Maximum cost horizon, $N_2 = 3$
- Control horizon , $N_u = 3$
- The control weighting coefficient, $\lambda = 0$
- T polynomial, and P numerator are not included.

The plant described as a CARIMA model, is:

$$(1 - 0.8z^{-1}) * y(t) = (0.4z^{-1} + 0.6z^{-2}) * u(t-1) + x(t),$$

where $x(t)$ is set to zero, assuming no disturbance signal, the model polynomials are as follows

$$A(z^{-1}) = 1 - 0.8z^{-1}$$

$$B(z^{-1}) = 0.4z^{-1} + 0.6z^{-2}$$

if

$$\Delta = 1 - z^{-1}$$

then

$$A * \Delta = \tilde{A} = (1 - 0.8z^{-1}) * (1 - z^{-1}) = 1 - 1.8z^{-1} + 0.8z^{-2}$$

Using the theory of Diophantine equation, the following equation can be solved

$$1 = E_j A\Delta + z^{-j} F_j z^{-1}$$

The solution may be obtained by long division of polynomials $\frac{1}{A\Delta}$, until the result can be factorized as $z^{-1}F_j$ the quotient of the division and the remainder are obtained below:

$$\begin{array}{lll}
\quad\quad\quad\quad\overline{1+1.8z^{-1}+2.44z^{-2}} & & \\
1-1.8z^{-1}\ +0.8z^{-2}\,\big)\,1 & & \\
\underline{1-1.8z^{-1}\ +0.8z^{-2}} & & \\
\quad 1.8z^{-1}\ -0.8z^{-2} & \Rightarrow E_1 = 1 & F_1 = 1.8 - 0.8z^{-1} \\
\quad \underline{1.8z^{-1}-3.24z^{-2}+1.44z^{-3}} & & \\
\quad\quad\quad 2.44z^{-2}-1.44z^{-3} & \Rightarrow E_2 = 1 + 1.8z^{-1} & F_2 = 2.44 - 1.44z^{-1} \\
\quad\quad\quad \underline{2.44z^{-2}-4.392z^{-3}+1.952z^{-4}} & & \\
\quad\quad\quad\quad\quad 2.952z^{-3}\ \ -1.952z^{-4} & \Rightarrow E_3 = 1 + 1.8z^{-1} + 2.44z^{-2} & \\
 & \quad F_3 = 2.952z^{-1} - 1.952z^{-2} &
\end{array}$$

The matrix $\mathbb{G}$ is obtained from the following expression

$$G'_j = E_j * B$$

$$G'_1 = 1 * (0.4z^{-1} + 0.6z^{-2}) = 0.4z^{-1} + 0.6z^{-2}$$

$$G'_2 = (1 + 1.8z^{-1}) * (0.4z^{-1} + 0.6z^{-2}) = 0.4z^{-1} + 1.32z^{-2} + 1.08z^{-3}$$

$$G'_3 = (1 + 1.8z^{-1} + 2.44z^{-2})*(0.4z^{-1} + 0.6z^{-2}) = 0.4z^{-1}+1.32z^{-2}+2.056z^{-3}+1.464z^{-4}$$

The quotient of the division and the remainder are below:

$$\begin{array}{ll}
G_1 = 0 & \bar{G}_1 = 0.4 + 0.6z^{-1} \\
G_2 = 0.4 & \bar{G}_2 = 1.32 + 1.08z^{-1} \\
G_3 = 0.4 + 1.32z^{-1} & \bar{G}_3 = 2.056 + 1.464z^{-1}
\end{array}$$

The dimension of matrix $\mathbb{G}$ is $N_2 \times N_u$, so 3×3 in this particular case.

$$\mathbb{F} = \begin{bmatrix} 1.8 & -0.8 \\ 2.44 & -1.44 \\ 2.952 & -1.952 \end{bmatrix} \quad \mathbb{G} = \begin{bmatrix} 0 & 0 & 0 \\ 0.4 & 0 & 0 \\ 1.32 & 0.4 & 0 \end{bmatrix} \quad \bar{\mathbb{G}} = \begin{bmatrix} 0.4 & 0.6 \\ 1.32 & 1.08 \\ 2.056 & 1.464 \end{bmatrix}$$

The $\mathbb{H}$ matrix is calculated from the equation:$\mathbb{H} = (\mathbb{G}^T\mathbb{G} + \lambda\mathbb{I})^{-1} * \mathbb{G}^T$, hence

$$\mathbb{H} = \left(\begin{bmatrix} 0 & 0.4 & 1.32 \\ 0 & 0 & 0.4 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 0 \\ 0.4 & 0 & 0 \\ 1.32 & 0.4 & 0 \end{bmatrix} + 0 * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right)^{-1} * \begin{bmatrix} 0 & 0.4 & 1.32 \\ 0 & 0 & 0.4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbb{H} = \begin{bmatrix} 0 & 2.5 & 0 \\ 0 & -8.25 & 2.5 \\ 0 & 0 & 0 \end{bmatrix}$$

$h = \sum$ first row of $\mathbb{H}$, in this case $h = 2.5$
S polynomial is the first row of $\mathbb{H} * \mathbb{F}$

$$\mathbb{H} * \mathbb{F} = \begin{bmatrix} 0 & 2.5 & 0 \\ 0 & -8.25 & 2.5 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1.8 & -0.8 \\ 2.44 & -1.44 \\ 2.952 & -1.952 \end{bmatrix} = \begin{bmatrix} 6.1 & -3.6 \\ -12.75 & 7 \\ 0 & 0 \end{bmatrix}$$

hence the S polynomial $S = 6.1 - 3.6z^{-1}$
R polynomial is the first row of $\mathbb{H} * \bar{\mathbb{G}}$

$$\mathbb{H} * \bar{\mathbb{G}} \begin{bmatrix} 0 & 2.5 & 0 \\ 0 & -8.25 & 2.5 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0.4 & 0.6 \\ 1.32 & 1.08 \\ 2.056 & 1.464 \end{bmatrix} = \begin{bmatrix} 3.3 & 2.7 \\ -5.75 & -5.25 \\ 0 & 0 \end{bmatrix}$$

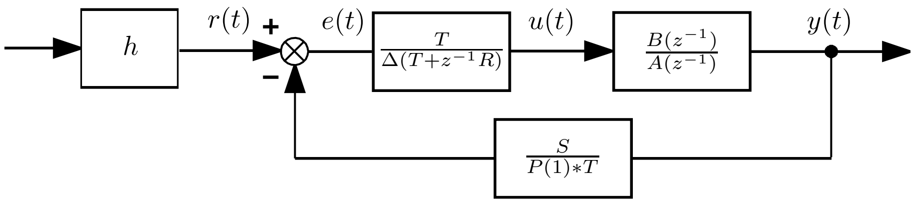

Figure A.1: The block diagram implementation of the GPC controller

the R polynomial $R = 3.3 + 2.7 * z^{-1}$. Implementation of the GPC controller is shown on figure A.1, where $P(1)$ denotes the static gain of the P polynomial.

Appendix B

Results

B.1 The PI controller

Process	Cost	K_p	K_i	IAE	M_p	M_s	GM	PM	DM	MM	Disturbance			Step response			
	function				(abs)	(abs)	(dB)	(deg)	(sec)	(abs)	T_{rd}	y_d	IAE	IAE	T_r	y_p	T_s
$G_1(s)$	IAE at response	1.573	0.500	4.389	1.19	1.81	3.54	53	1.39	0.55	13.0	0	2	2.4	1.9	12.9	9.6
	IAE at disturbance	3.429	0.899	4.812	3.18	3.65	1.74	20	0.31	0.27	13.9	7	1.3	3.7	1.1	45.6	23.3
	both	2.044	0.674	4.096	1.63	2.23	2.69	39	0.82	0.45	11.3	0.45	1.5	2.6	1.5	26.4	13.9
$G_2(s)$	IAE at response	5.340	3.992	0.684	1.36	1.81	4.84	43	0.19	0.55	3.3	0	0.25	0.4	0.3	23.4	1.7
	IAE at disturbance	14.687	15.606	0.860	5.00	5.33	1.63	12	0.03	0.19	2.1	6.9	0.072	0.8	0.1	71.1	4.8
	both	7.485	6.905	0.618	1.89	2.32	3.31	31	0.11	0.43	2.1	0	0.14	0.5	0.2	39.9	2.4

Process	Cost	K_p	K_i	IAE	M_p	M_s	GM	PM	DM	MM	Disturbance			Step response			
	function				(abs)	(abs)	(dB)	(deg)	(sec)	(abs)	T_{rd}	y_d	IAE	IAE	T_r	y_p	T_s
$G_3(s)$	IAE at response	0.367	0.043	48.373	1.00	1.92	2.12	64	24.51	0.52	92.2	3.2	24	24.2	13.8	3.3	74.1
	IAE at disturbance	0.374	0.043	48.373	1.00	1.93	2.10	65	24.55	0.52	92.1	3.2	24	24.2	13.6	3.3	74.0
	both	0.372	0.043	48.371	1.00	1.93	2.11	64	24.54	0.52	92.2	3.2	24	24.2	13.7	3.3	74.0
$G_4(s)$	IAE at response	0.472	0.001	41.450	1.30	1.79	4.22	46	1.96	0.56	40.0	0	78	4.0	3.0	23.5	15.7
	IAE at disturbance	1.122	0.105	16.164	6.06	6.44	1.45	10	0.24	0.16	39.5	9.6	11	9.8	1.5	81.4	39.4
	both	1.237	0.132	16.004	9.64	10.03	1.27	6	0.14	0.10	40.0	22	12	12.9	1.4	88.7	40.0
$G_5(s)$	IAE at response	0.455	0.177	12.696	1.22	2.15	1.92	61	5.26	0.47	22.0	32	6.8	5.9	3.0	5.0	17.4
	IAE at disturbance	0.484	0.185	12.666	1.36	2.28	1.82	59	4.78	0.44	21.6	32	6.7	5.9	2.7	9.7	16.9
	both	0.468	0.182	12.641	1.29	2.21	1.87	60	5.00	0.45	21.8	32	6.8	5.9	2.9	7.5	17.2
$G_6(s)$	IAE at response	0.755	1.076	2.092	1.00	1.85	2.33	66	1.13	0.54	5.4	4.7	0.99	1.1	0.9	5.6	4.6
	IAE at disturbance	0.784	1.336	2.060	1.13	2.06	2.06	58	0.85	0.49	5.6	9	0.9	1.2	0.8	17.6	5.6
	both	0.773	1.218	2.036	1.03	1.96	2.17	61	0.96	0.51	5.5	6.6	0.91	1.1	0.9	12.0	4.7
$G_7(s)$	IAE at response	0.343	0.768	2.707	1.00	1.92	2.13	63	1.35	0.52	5.0	11	1.3	1.4	0.8	11.1	4.1
	IAE at disturbance	0.327	0.761	2.695	1.00	1.88	2.18	63	1.36	0.53	4.6	8.9	1.3	1.4	0.8	8.8	4.1
	both	0.327	0.755	2.680	1.00	1.87	2.19	63	1.38	0.53	4.6	8	1.3	1.4	0.8	8.1	4.0
$G_8(s)$	IAE at response	0.637	0.001	91.031	1.14	1.80	2.47	53	1.46	0.56	100.0	0	inf	2.3	1.4	15.2	5.5
	IAE at disturbance	0.919	0.224	9.041	3.13	3.75	1.50	21	0.39	0.27	15.6	8.6	4.9	4.1	0.8	83.4	15.5
	both	0.831	0.204	8.752	2.55	3.17	1.65	25	0.50	0.32	16.1	3.6	5	3.7	0.8	75.1	13.2

Process	Cost	K_p	K_i	IAE	M_p	M_s	GM	PM	DM	MM	Disturbance			Step response			
	function				(abs)	(abs)	(dB)	(deg)	(sec)	(abs)	T_{rd}	y_d	IAE	IAE	T_r	y_p	T_s
$G_{10}(s)$	IAE at response	4.492	2.612	0.892	1.69	2.19	3.04	36	0.12	0.46	4.3	0.068	0.38	0.5	0.2	35.3	2.9
	IAE at disturbance	7.252	8.675	0.904	4.26	4.65	1.65	14	0.04	0.22	2.2	7	0.13	0.8	0.2	69.9	4.7
	both	4.633	5.901	0.725	2.22	2.67	2.54	27	0.09	0.37	1.9	0.11	0.17	0.6	0.2	52.5	2.9
$G_{11}(s)$	IAE at response	3.160	2.685	0.761	1.24	1.75	4.35	48	0.21	0.57	3.5	0	0.37	0.4	0.3	19.6	1.6
	IAE at disturbance	7.567	9.618	0.841	4.11	4.50	1.66	15	0.04	0.22	2.2	7.1	0.12	0.7	0.2	65.9	4.1
	both	4.433	5.532	0.639	1.88	2.35	2.85	32	0.11	0.43	1.9	0	0.18	0.5	0.2	41.7	2.3
$G_{12}(s)$	IAE at response	0.834	0.001	116.797	1.56	1.94	7.23	38	1.07	0.52	200.0	0	inf	3.4	2.0	31.8	15.6
	IAE at disturbance	2.476	0.245	13.486	7.76	7.98	1.74	8	0.12	0.13	42.0	6.8	4.7	8.8	1.0	77.6	55.4
	both	1.524	0.185	11.338	3.99	4.20	2.61	15	0.29	0.24	32.3	1.7	5.5	5.8	1.3	69.6	32.2
$G_{13}(s)$	IAE at response	99.776	0.001	128.920	1299774.65	1299774.65	Inf	0	0.00	0.00	199.5	0	2	128.6	0.1	102.1	200.0
	IAE at disturbance	99.789	0.001	128.885	1300013.99	1300013.99	Inf	0	0.00	0.00	199.5	0	2	128.6	0.1	102.1	200.0
	both	999.02	0.999	65	31592	31592.28	Inf	0	0.00	0.00	199.5	0	0.096	65.30	0.026	105	99.99
$G_{14}(s)$	IAE at response	999.992	100.013	0.020	1.00	1.00	Inf	90	0.02	1.00	0.0	0.042	0.01	0.0	0.0	0.0	0.0
	IAE at disturbance	196.066	999.992	0.075	1.15	1.00	Inf	76	0.07	1.00	0.0	0	0.001	0.1	0.1	13.5	0.5
	both	999.024	0.999	0.019	1.00	1.00	Inf	90	0.02	1.00	0.0	0	0.26	0.3	0.0	-0.1	0.0

B.2 The GPC controller — fixed control horizon

No of	Cost	T_s	N_1	N_2	N_u	λ	T polynomial	IAE	M_p	M_s	GM	PM	DM	MM	Disturbance			Step response			
system	function								(abs)	(abs)	(dB)	(deg)	(sec)	(abs)	T_{rd}	y_d	IAE	IAE	T_r	y_p	T_s
$G_1(s)$	IAE at response	0.70	1	2	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	1.33	1.00	3.95	2.59	21	0.20	0.25	3.0	7.5	0.28	1.1	1.0	8.0	2.9
	IAE at disturbance	0.70	1	2	1	$1*10^{-6}$	$(1+0.342z^{-1})(1+0.336z^{-1})(1+0.399z^{-1})$	1.23	1.00	9.35	-0.93	-6	0.09	0.11	3.1	38	0.18	1.1	1.0	8.0	2.9
	both	0.70	1	2	1	$1*10^{-6}$	$(1+0.341z^{-1})(1+0.392z^{-1})(1+0.34z^{-1})$	1.23	1.00	9.29	-0.94	-6	0.09	0.11	3.1	37	0.18	1.1	1.0	8.0	2.9
$G_2(s)$	IAE at response	0.10	1	2	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.163	1.00	4.35	2.32	18	0.02	0.23	0.4	6.2	0.011	0.2	0.1	7.0	0.4
	IAE at disturbance	0.10	1	2	1	$1*10^{-6}$	$(1+0.327z^{-1})(1+0.343z^{-1})(1+0.319z^{-1})$	0.159	1.00	9.39	-0.93	-6	0.01	0.11	0.3	33	0.0072	0.2	0.1	7.0	0.4
	both	0.10	1	2	1	$1*10^{-6}$	$(1+0.326z^{-1})(1+0.335z^{-1})(1+0.334z^{-1})$	0.159	1.00	9.46	-0.92	-6	0.01	0.11	0.3	34	0.0072	0.2	0.1	7.0	0.4
$G_3(s)$	IAE at response	0.70	23	23	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	30	1.00	14.05	0.64	-4	0.02	0.07	33.5	3.5	14	15.9	0.7	11.2	17.1
	IAE at disturbance	0.70	22	23	1	$1*10^{-6}$	$(1+0.673z^{-1})(1-0.0986z^{-1})(1-0.0156z^{-1})$	29.8	1.00	46.98	0.19	1	0.00	0.02	33.0	9.5	14	15.9	0.7	15.8	17.9
	both	0.70	23	23	1	$1*10^{-6}$	$(1-0.00781z^{-1})(1-0.0176z^{-1})(1+0.635z^{-1})$	29.8	1.00	44.19	0.20	1	0.01	0.02	33.0	10	14	15.9	0.7	11.2	17.1
$G_4(s)$	IAE at response	0.20	2	2	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.391	1.03	9.23	1.08	7	0.02	0.11	0.0	17	0.0072	0.4	0.3	12.0	1.3
	IAE at disturbance	0.20	2	2	1	$1*10^{-6}$	$(1+0.483z^{-1})(1+0.486z^{-1})(1+0.524z^{-1})$	0.388	1.03	47.10	-0.18	-1	0.01	0.02	0.0	42	0.0038	0.4	0.3	12.0	1.3
	both	0.20	2	2	1	$1*10^{-6}$	$(1+0.502z^{-1})(1+0.49z^{-1})(1+0.501z^{-1})$	0.388	1.03	46.90	-0.18	-1	0.01	0.02	0.0	42	0.0038	0.4	0.3	12.0	1.3
$G_5(s)$	IAE at response	0.15	8	51	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	8.64	1.02	15.32	0.73	-4	0.15	0.07	9.8	100	3.7	4.9	3.7	0.0	8.9
	IAE at disturbance	0.15	8	48	1	$1*10^{-6}$	$(1+0.737z^{-1})(1+0.736z^{-1})(1+0.737z^{-1})$	8.36	1.02	756.64	-0.01	0	0.12	0.00	9.5	140	3.4	4.9	3.7	0.0	8.9
	both	0.15	8	50	1	$1*10^{-6}$	$(1+0.73z^{-1})(1+0.738z^{-1})(1+0.733z^{-1})$	8.36	1.02	721.85	-0.01	0	0.12	0.00	9.5	140	3.4	4.9	3.7	0.0	8.9
$G_6(s)$	IAE at response	0.10	2	2	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.197	1.04	9.47	1.05	7	0.01	0.11	0.0	17	0.0043	0.2	0.1	15.4	0.7

No of	Cost	T_s	N_1	N_2	N_u	λ	T polynomial	IAE	M_p	M_s	GM	PM	DM	MM	Disturbance			Step response			
system	function								(abs)	(abs)	(dB)	(deg)	(sec)	(abs)	T_{rd}	y_d	IAE	IAE	T_r	y_p	T_s
	IAE at disturbance	0.10	2	2	1	$1*10^{-6}$	$(1+0.596z^{-1})(1+0.438z^{-1})(1+0.478z^{-1})$	0.195	1.04	52.31	-0.16	-1	0.00	0.02	0.0	44	0.0023	0.2	0.1	15.4	0.7
	both	0.10	2	2	1	$1*10^{-6}$	$(1+0.499z^{-1})(1+0.505z^{-1})(1+0.511z^{-1})$	0.195	1.04	51.25	-0.17	-1	0.00	0.02	0.0	44	0.0023	0.2	0.1	15.4	0.7
$G_7(s)$	IAE at response	0.10	23	55	1	0.078	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	2.1	1.00	2.00	6.02	60	0.04	0.50	2.1	0	1.1	1.1	0.0	99.5	2.1
	IAE at disturbance	0.10	10	73	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	2.1	1.00	2.00	6.02	-60	0.04	0.50	2.1	0	1.1	1.1	0.0	100.0	2.1
	both	0.10	10	73	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	2.1	1.00	2.00	6.02	-60	0.04	0.50	2.1	0	1.1	1.1	0.0	100.0	2.1
$G_8(s)$	IAE at response	0.10	10	11	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	1.71	1.00	23.99	0.37	2	0.00	0.04	2.2	0.02	0.66	1.1	0.1	0.0	1.1
	IAE at disturbance	0.10	10	11	1	$1*10^{-6}$	$(1+0z^{-1})(1+0.00781z^{-1})(1+0z^{-1})$	1.71	1.00	24.34	0.36	2	0.00	0.04	2.2	0.73	0.66	1.1	0.1	0.0	1.1
	both	0.10	10	11	1	$1*10^{-6}$	$(1+0.00391z^{-1})(1-0.000977z^{-1})(1-0.000977z^{-1})$	1.71	1.00	24.08	0.37	2	0.00	0.04	2.2	0.14	0.66	1.1	0.1	0.0	1.1
$G_{10}(s)$	IAE at response	1.00	1	1	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	2.03	1.00	5.34	1.80	17	0.15	0.19	3.3	65	1.4	0.6	0.7	12.8	2.5
	IAE at disturbance	1.00	1	2	1	$1*10^{-6}$	$(1+0.192z^{-1})(1+0.221z^{-1})(1-0.675z^{-1})$	2.17	1.00	3.53	2.89	32	0.34	0.28	3.6	3.9	1.1	1.1	1.7	-0.4	5.0
	both	1.00	1	1	1	$1*10^{-6}$	$(1-0.0234z^{-1})(1-0.693z^{-1})(1+0.103z^{-1})$	1.73	1.00	3.43	2.99	31	0.33	0.29	2.8	5.3	1.1	0.6	0.7	12.8	2.5
$G_{11}(s)$	IAE at response	0.10	1	2	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.172	1.00	4.32	2.36	18	0.02	0.23	0.4	7.4	0.019	0.2	0.1	7.8	0.4
	IAE at disturbance	0.10	1	2	1	$1*10^{-6}$	$(1+0.367z^{-1})(1+0.363z^{-1})(1+0.367z^{-1})$	0.165	1.00	10.41	-0.82	-6	0.01	0.10	0.4	37	0.012	0.2	0.1	7.8	0.4
	both	0.10	1	2	1	$1*10^{-6}$	$(1+0.373z^{-1})(1+0.36z^{-1})(1+0.369z^{-1})$	0.165	1.00	10.50	-0.81	-5	0.01	0.10	0.4	37	0.012	0.2	0.1	7.8	0.4
$G_{12}(s)$	IAE at response	0.20	1	6	3	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.168	1.00	8.82	1.06	7	0.01	0.11	0.0	37	0.0033	0.2	0.2	17.5	0.8
	IAE at disturbance	0.20	1	3	3	$1*10^{-6}$	$(1-0.337z^{-1})(1-0.326z^{-1})(1+0.453z^{-1})$	0.168	1.00	8.83	1.04	9	0.01	0.11	0.0	13	0.0027	0.2	0.1	22.5	0.8
	both	0.20	1	6	3	$1*10^{-6}$	$(1-0.278z^{-1})(1+0.433z^{-1})(1-0.355z^{-1})$	0.167	1.00	8.77	1.05	9	0.01	0.11	0.0	13	0.0027	0.2	0.2	17.5	0.8
$G_{13}(s)$	IAE at response	0.10	1	2	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.161	1.00	3.63	3.47	18	0.02	0.28	0.0	6.8	0.0034	0.2	0.2	6.8	0.5

No of	Cost	T_s	N_1	N_2	N_u	λ	T polynomial	IAE	M_p	M_s	GM	PM	DM	MM	Disturbance			Step response			
system	function								(abs)	(abs)	(dB)	(deg)	(sec)	(abs)	T_{rd}	y_d	IAE	IAE	T_r	y_p	T_s
	IAE at disturbance	0.10	1	2	1	$1*10^{-6}$	$(1+0.78z^{-1})(1+0.308z^{-1})(1+0.543z^{-1})$	0.159	1.00	10.33	0.90	9	0.01	0.10	0.0	12	0.0013	0.2	0.2	6.8	0.5
	both	0.10	1	2	1	$1*10^{-6}$	$(1+0.389z^{-1})(1+0.407z^{-1})(1+1z^{-1})$	0.159	1.00	12.08	0.75	9	0.01	0.08	0.0	11	0.0011	0.2	0.2	6.8	0.5
$G_{14}(s)$	IAE at response	1.00	1	1	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.587	1.00	3.81	2.65	26	0.26	0.26	1.6	0	0.095	0.5	0.8	0.0	0.9
	IAE at disturbance	1.00	1	1	1	$1*10^{-6}$	$(1+0.0313z^{-1})(1-0.0234z^{-1})(1+0.00586z^{-1})$	0.587	1.00	3.86	2.60	25	0.25	0.26	1.6	1.3	0.095	0.5	0.8	0.0	0.9
	both	1.00	1	1	1	$1*10^{-6}$	$(1-0.0254z^{-1})(1+0.0488z^{-1})(1-0.0107z^{-1})$	0.587	1.00	3.86	2.60	25	0.25	0.26	1.6	1.2	0.095	0.5	0.8	0.0	0.9

B.3 The GPC controller — variable control horizon

No of	Cost	T_s	N_1	N_2	N_u	λ	T polynomial	IAE	M_p	M_s	GM	PM	DM	MM	Disturbance			Step response			
system	function								(abs)	(abs)	(dB)	(deg)	(sec)	(abs)	T_{rd}	y_d	IAE	IAE	T_r	y_p	T_s
$G_1(s)$	IAE at response	0.70	1	6	3	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	1.04	1.00	5.83	1.64	-12	0.14	0.17	2.6	5.7	0.19	0.8	0.7	12.6	2.4
	IAE at disturbance	0.70	2	55	8	$1*10^{-6}$	$(1+0.267z^{-1})(1+0.156z^{-1})(1+0.204z^{-1})$	1.04	0.95	10.71	-0.84	-5	0.08	0.09	3.0	38	0.18	0.9	0.8	3.7	1.9
	both	0.70	1	6	3	$1*10^{-6}$	$(1+0.085z^{-1})(1+0.333z^{-1})(1+0.0322z^{-1})$	1.02	1.00	9.20	-1.02	-6	0.09	0.11	3.0	31	0.18	0.8	0.7	12.6	2.4
$G_2(s)$	IAE at response	0.10	1	7	3	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.131	1.00	6.23	1.53	-11	0.02	0.16	0.3	5.6	0.0077	0.1	0.1	12.4	0.3
	IAE at disturbance	0.10	1	26	5	$1*10^{-6}$	$(1+0.0244z^{-1})(1+0.149z^{-1})(1+0.0498z^{-1})$	0.134	1.00	8.23	1.17	-7	0.01	0.12	0.3	21	0.0071	0.1	0.1	21.9	0.4
	both	0.10	1	7	3	$1*10^{-6}$	$(1+0.0771z^{-1})(1+0.0107z^{-1})(1+0.327z^{-1})$	0.131	1.00	9.37	-0.99	-6	0.01	0.11	0.3	30	0.0071	0.1	0.1	12.4	0.3
$G_3(s)$	IAE at response	0.70	23	71	4	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	29.9	1.00	17.22	0.52	-4	0.01	0.06	32.8	0.81	14	15.9	0.6	12.7	17.4
	IAE at disturbance	0.70	22	67	12	$1*10^{-6}$	$(1-0.0156z^{-1})(1-0.0283z^{-1})(1+0.263z^{-1})$	29.8	1.00	42.45	0.21	1	0.01	0.02	33.5	9.7	14	16.0	0.6	24.9	18.7
	both	0.70	23	79	4	$1*10^{-6}$	$(1+0.0156z^{-1})(1-0.00684z^{-1})(1+0.416z-1)$	29.7	1.00	40.22	0.22	1	0.01	0.02	33.4	8.4	14	15.9	0.6	12.7	17.4

No of	Cost	T_s	N_1	N_2	N_u	λ	T polynomial	IAE	M_p	M_s	GM	PM	DM	MM	Disturbance			Step response			
system	function								(abs)	(abs)	(dB)	(deg)	(sec)	(abs)	T_{rd}	y_d	IAE	IAE	T_r	y_p	T_s
$G_4(s)$	IAE at response	0.20	1	128	4	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.326	1.00	11.06	0.83	7	0.01	0.09	0.0	7.9	0.0054	0.3	0.3	4.9	0.8
	IAE at disturbance	0.20	2	128	4	$1*10^{-6}$	$(1+0.352z^{-1})(1+0.366z^{-1})(1+0.385z^{-1})$	0.33	1.00	16.80	-0.27	-2	0.01	0.06	0.0	35	0.0035	0.3	0.3	4.6	0.8
	both	0.20	1	128	4	$1*10^{-6}$	$(1+0.273z^{-1})(1+0.597z^{-1})(1+0.108z^{-1})$	0.324	1.00	16.28	-0.26	-2	0.01	0.06	0.0	33	0.0035	0.3	0.3	4.9	0.8
$G_5(s)$	IAE at response	0.15	2	58	6	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	11	10.34	4269.08	0.00	-0	NaN	0.00	2.8	71	8.8	2.3	0.0	200.1	1.3
	IAE at disturbance	0.15	6	51	2	0.31	$(1+0.784z^{-1})(1+0.784z^{-1})(1+0.784z^{-1})$	7.21	1.47	1026.27	-0.01	-0	0.05	0.00	5.7	130	3.3	3.9	1.8	0.0	5.1
	both	0.15	2	36	7	$1*10^{-6}$	$(1-0.855z^{-1})(1-0.855z^{-1})(1+0.938z^{-1})$	5.52	10.89	379.95	-0.02	-0	0.01	0.00	7.5	210	3.2	2.3	0.0	312.1	1.4
$G_6(s)$	IAE at response	0.10	1	64	4	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.157	1.00	11.88	0.76	-6	0.01	0.08	0.0	5.8	0.003	0.2	0.1	15.5	0.5
	IAE at disturbance	0.10	2	64	4	$1*10^{-6}$	$(1+0.187z^{-1})(1+0.389z^{-1})(1+0.397z^{-1})$	0.159	1.00	23.18	-0.37	-3	0.00	0.04	0.0	22	0.0021	0.2	0.1	15.8	0.4
	both	0.10	1	64	4	$1*10^{-6}$	$(1+0.272z^{-1})(1+0.551z^{-1})(1+0.123z^{-1})$	0.156	1.00	35.83	-0.24	-2	0.00	0.03	0.0	36	0.002	0.2	0.1	15.5	0.5
$G_7(s)$	IAE at response	0.10	14	46	1	0.078	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	2.1	1.00	2.00	6.02	60	0.04	0.50	2.1	0	1.1	1.1	0.0	99.5	2.1
	IAE at disturbance	0.10	10	73	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	2.1	1.00	2.00	6.02	-60	0.04	0.50	2.1	0	1.1	1.1	0.0	100.0	2.1
	both	0.10	10	73	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	2.1	1.00	2.00	6.02	-60	0.04	0.50	2.1	0	1.1	1.1	0.0	100.0	2.1
$G_8(s)$	IAE at response	0.10	14	59	2	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	1.71	1.00	24.00	0.37	2	0.00	0.04	2.2	0.021	0.66	1.1	0.1	0.0	1.1
	IAE at disturbance	0.10	11	33	2	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0.000977z^{-1})$	1.71	1.00	24.04	0.37	2	0.00	0.04	2.2	0.051	0.66	1.1	0.1	0.0	1.1
	both	0.10	10	27	16	$1*10^{-6}$	$(1+0z^{-1})(1+0.000977z^{-1})(1+0.000977z^{-1})$	1.71	1.00	24.07	0.37	2	0.00	0.04	2.2	0.12	0.66	1.1	0.1	0.0	1.1
$G_{10}(s)$	IAE at response	1.00	1	22	8	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	2.03	1.00	5.33	1.81	18	0.15	0.19	3.3	64	1.4	0.6	0.7	12.9	2.5
	IAE at disturbance	1.00	2	29	5	0.078	$(1+0.185z^{-1})(1+0.183z^{-1})(1+0.181z^{-1})$	2.44	1.03	3.75	2.70	31	0.32	0.27	5.0	9.6	1.2	1.3	1.7	5.9	6.8
	both	1.00	1	45	9	$1*10^{-6}$	$(1+0.0996z^{-1})(1-0.0244z^{-1})(1-0.693z^{-1})$	1.73	1.00	3.42	3.01	31	0.33	0.29	2.8	5	1.1	0.6	0.7	12.9	2.5

No of	Cost	T_s	N_1	N_2	N_u	λ	T polynomial	IAE	M_p	M_s	GM	PM	DM	MM	Disturbance			Step response			
system	function								(abs)	(abs)	(dB)	(deg)	(sec)	(abs)	T_{rd}	y_d	IAE	IAE	T_r	y_p	T_s
$G_{11}(s)$	IAE at response	0.10	1	32	4	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.135	1.00	7.28	1.31	-9	0.02	0.14	0.3	10	0.012	0.1	0.1	18.6	0.4
	IAE at disturbance	0.10	1	2	1	$1*10^{-6}$	$(1+0.381z^{-1})(1+0.345z^{-1})(1+0.369z^{-1})$	0.165	1.00	11.02	-0.77	-5	0.01	0.09	0.4	38	0.012	0.2	0.1	7.8	0.4
	both	0.10	1	32	4	$1*10^{-6}$	$(1+0.0459z^{-1})(1+0.083z^{-1})(1+0.222z^{-1})$	0.135	1.00	10.65	-0.86	-6	0.01	0.09	0.3	34	0.011	0.1	0.1	18.6	0.4
$G_{12}(s)$	IAE at response	0.20	1	6	3	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.168	1.00	8.82	1.06	7	0.01	0.11	0.0	37	0.0033	0.2	0.2	17.5	0.8
	IAE at disturbance	0.20	1	3	3	$1*10^{-6}$	$(1-0.337z^{-1})(1-0.326z^{-1})(1+0.453z^{-1})$	0.168	1.00	8.83	1.04	9	0.01	0.11	0.0	13	0.0027	0.2	0.1	22.5	0.8
	both	0.20	1	6	3	$1*10^{-6}$	$(1-0.278z^{-1})(1+0.433z^{-1})(1-0.355z^{-1})$	0.167	1.00	8.77	1.05	9	0.01	0.11	0.0	13	0.0027	0.2	0.2	17.5	0.8
$G_{13}(s)$	IAE at response	0.10	1	2	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.161	1.00	3.63	3.47	18	0.02	0.28	0.0	6.8	0.0034	0.2	0.2	6.8	0.5
	IAE at disturbance	0.10	1	2	1	$1*10^{-6}$	$(1+0.78z^{-1})(1+0.308z^{-1})(1+0.543z^{-1})$	0.159	1.00	10.33	0.90	9	0.01	0.10	0.0	12	0.0013	0.2	0.2	6.8	0.5
	both	0.10	1	2	1	$1*10^{-6}$	$(1+0.389z^{-1})(1+0.407z^{-1})(1+1z^{-1})$	0.159	1.00	12.08	0.75	9	0.01	0.08	0.0	11	0.0011	0.2	0.2	6.8	0.5
$G_{14}(s)$	IAE at response	1.00	1	1	1	$1*10^{-6}$	$(1+0z^{-1})(1+0z^{-1})(1+0z^{-1})$	0.587	1.00	3.81	2.65	26	0.26	0.26	1.6	0	0.095	0.5	0.8	0.0	0.9
	IAE at disturbance	1.00	1	1	1	$1*10^{-6}$	$(1+0.0313z^{-1})(1-0.0234z^{-1})(1+0.00586z-1)$	0.587	1.00	3.86	2.60	25	0.25	0.26	1.6	1.3	0.095	0.5	0.8	0.0	0.9
	both	1.00	1	1	1	$1*10^{-6}$	$(1-0.0254z^{-1})(1+0.0488z^{-1})(1-0.0107z-1)$	0.587	1.00	3.86	2.60	25	0.25	0.26	1.6	1.2	0.095	0.5	0.8	0.0	0.9

B.4 The GPC controller — including P numerator

No of	T_s	N_1	N_2	N_u	λ	P numerator	T polynomial	IAE	M_p	M_s	GM	PM	DM	MM	Disturbance			Step response			
system									(abs)	(abs)	(dB)	(deg)	(sec)	(abs)	T_{rd}	y_d	IAE	IAE	T_r	y_p	T_s
$G_1(s)$	0.70	2	11	6	$1*10^{-6}$	$(1+0.138z^{-1})$	$(1+0.188z^{-1})(1+0.251z^{-1})(1+0.178z^{-1})$	1.07	1.00	8.58	-1.12	-7	0.09	0.12	3.0	25	0.18	0.9	0.8	0.8	1.5
$G_2(s)$	0.10	1	7	3	$1*10^{-6}$	$(1-0.0752z^{-1})$	$(1+0.103z^{-1})(1-0.00879z^{-1})(1+0.321z^{-1})$	0.133	1.00	6.31	1.51	-11	0.02	0.16	0.3	3.5	0.0077	0.1	0.1	19.3	0.4
$G_3(s)$	0.70	23	84	4	$1*10^{-6}$	$(1+0.612z^{-1})$	$(1-0.0322z^{-1})(1+0.0293z^{-1})(1+0.435z^{-1})$	29.8	1.00	45.02	0.20	1	0.01	0.02	33.5	11	14	15.9	0.6	16.3	17.4

No of	T_s	N_1	N_2	N_u	λ	P numerator	T polynomial	IAE	M_p	M_s	GM	PM	DM	MM	Disturbance			Step response			
system									(abs)	(abs)	(dB)	(deg)	(sec)	(abs)	T_{rd}	y_d	IAE	IAE	T_r	y_p	T_s
$G_4(s)$	0.20	1	128	4	$1 * 10^{-6}$	$(1 + 0.361z^{-1})$	$(1 + 0.485z^{-1})(1 + 0.00391z^{-1})(1 + 0.498z^{-1})$	0.324	1.00	13.64	-0.36	-3	0.01	0.07	0.0	8.8	0.0036	0.3	0.3	4.9	0.8
$G_5(s)$	0.15	5	29	3	$1 * 10^{-6}$	$(1 - 0.783z^{-1})$	$(1 + 0.645z^{-1})(1 - 0.921z^{-1})(1 + 0.647z^{-1})$	5.76	5.66	1198.19	-0.01	-0	NaN	0.00	8.6	290	3.3	2.4	0.6	0.0	1.9
$G_6(s)$	0.10	1	64	4	$1 * 10^{-6}$	$(1 - 0.853z^{-1})$	$(1 + 0.443z^{-1})(1 + 0.492z^{-1})(1 + 0.0137z^{-1})$	0.159	1.00	23.35	-0.36	-3	0.01	0.04	0.0	9.3	0.0021	0.2	0.1	15.8	0.4
$G_7(s)$	0.10	10	73	1	$1 * 10^{-6}$	$(1 - 0.312z^{-1})$	$(1 + 0z^{-1})(1 + 0z^{-1})(1 + 0z^{-1})$	2.1	1.00	2.00	6.02	60	0.04	0.50	2.1	0	1.1	1.1	0.0	99.6	2.1
$G_8(s)$	0.10	11	42	13	$1 * 10^{-6}$	$(1 - 0.136z^{-1})$	$(1 - 0.000977z^{-1})(1 + 0z^{-1})(1 + 0.000977z^{-1})$	1.73	1.00	20.32	0.44	3	0.00	0.05	2.3	0.019	0.67	1.1	0.1	0.0	1.2
$G_{10}(s)$	1.00	1	45	9	$1 * 10^{-6}$	$(1 - 0.883z^{-1})$	$(1 - 0.0244z^{-1})(1 - 0.693z^{-1})(1 + 0.0996z^{-1})$	2.51	1.02	4.14	2.40	31	0.31	0.24	5.0	14	1.2	1.3	1.8	5.6	6.4
$G_{11}(s)$	0.10	1	32	4	$1 * 10^{-6}$	$(1 + 0.648z^{-1})$	$(1 + 0.0664z^{-1})(1 + 0.0645z^{-1})(1 + 0.22z^{-1})$	0.165	1.00	8.68	-1.05	-7	0.01	0.12	0.3	30	0.012	0.2	0.1	7.8	0.4
$G_{12}(s)$	0.20	1	18	16	$1 * 10^{-6}$	$(1 + 0.877z^{-1})$	$(1 - 0.365z^{-1})(1 - 0.292z^{-1})(1 + 0.447z^{-1})$	0.169	1.00	14.68	0.61	7	0.01	0.07	0.0	37	0.0033	0.2	0.1	22.4	0.8
$G_{13}(s)$	0.10	1	16	3	$1 * 10^{-6}$	$(1 + 0.405z^{-1})$	$(1 + 0.131z^{-1})(1 + 0.146z^{-1})(1 + 1z^{-1})$	0.268	1.00	inf	0.00	NaN	0.10	0.00	0.0	100	0.38	0.1	0.1	1.1	0.2
$G_{14}(s)$	1.00	1	256	2	$1 * 10^{-6}$	$(1 - 0.933z^{-1})$	$(1 + 0.00781z^{-1})(1 + 0.000977z^{-1})(1 + 0.00488z^{-1})$	0.587	1.00	3.81	2.64	26	0.26	0.26	1.6	0	0.095	0.5	0.8	0.0	0.9