Intelligent controller design based on gain and phase margin specifications

Daniel Czarkowski†[1] **and Tom O'Mahony***

Advanced Control Group
Department of Electronic Engineering
Cork Institute of Technology
IRELAND

E-mail: †`dczarkowski@cit.ie` *`tomahony@cit.ie`

Abstract — **This paper attempts to determine whether advanced control algorithms offer any real benefits to the process industry in terms of optimising SISO process units. To establish this, a classical PID controller, Two Degree of Freedom (2–DOF) PID controller and the Generalised Predictive Controller, GPC, are compared. The same controller design philosophy is applied to all controllers and the performance is evaluated on a number of benchmark systems. Within this framework the GPC performs notably better than the other two counterparts.**

Keywords — **PID controller, 2–DOF, GPC, Genetic Algorithm, gain and phase margin.**

I Introduction

Over the past few decades, control engineers, (particulary academics) have devised a bewildering range of control algorithms to overcome the limitations of the classical PID controller. Frequently these proposed techniques are justified based on superior simulation performance with a PID controller. It is the author's contention that in many cases, these claims are unjustified because the PID controller was poorly designed and tuned, e.g. using the Ziegler–Nichols technique. Thus, the contribution of this paper is to compare an appropriately designed and well tuned PID controller with an advanced controller algorithm. The motivation for this study is to determine whether the advanced control algorithm can offer any real benefits to the process industry in terms of optimising process units by, for example, maximising throughput, minimising waste, minimising energy, etc.

As a representative advanced control algorithm the *Model–Based Predictive Control*, (MBPC), paradigm was chosen since MBPC is one control technique that has made a significant impact on the process industry. The suitability of MBPC for large multivariable constrained problems is not disputed. However, for single–input single–output systems the authors believe that PID may achieve a similar level of performance to MBPC, if the same amount of time, effort and expertise is invested. In an effort to establish whether this is indeed true, the performance of a *Two Degree of Freedom PID Controller*, (2–DOF PID), will be compared with that of the *Generalised Predictive Controller*, (GPC) [1]. Performance is compared over a range of classical and a–typical process transfer function models [2, 3] to enable general conclusions to be drawn. To ensure a fair comparison, both controllers are tuned to optimise the same criterion. An intelligent (i.e. from the field of Intelligent Systems) controller design philosophy is applied where the optimisation problem is defined as minimising the integral of error criterion subject to constraints on the gain and phase margins. This problem is solved numerically via a genetic algorithm. A benchmark is provided by a similarly tuned classical PID controller structure.

[1] This research was funded by the Irish Government under the HEAPRTLI cycle II scheme and part financed by the European Union ERDF. The authors wish to acknowledge this support.

II Controller description

As mentioned in the introduction three controller structures will be compared. The classical PID controller, 2–DOF PID controller, also called the industrial PID controller [4], and the GPC controller. Section II(a) describes the PID control laws while the GPC is presented in sec. II(b).

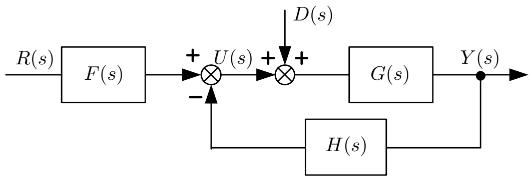


Fig. 1: Block diagram of 2–DOF PID controller

II(a) PID controllers

The classical PID controller is given by equation 1

$$U(s) = E(s)\left(K_p + \frac{K_i}{s} + K_d s\right) \tag{1}$$

where K_p, K_i and K_d are controller parameters, $U(s)$ is the control signal and the $E(s)$ is the control error. Equation 1 is known as a single degree of freedom control law because the controller parameters may be optimised for servo performance or regulator performance but not both. Most industrial controllers are considerably more complex than the "textbook" control law of equation 1; a typical representation is given by equation 2.

$$U(s) = K_p\left(bR(s) - Y(s)\right) + \frac{K_i}{s}\left(R(s) - Y(s)\right) + \frac{sK_d\left(cR(s) - Y(s)\right)}{1 + \frac{sK_d}{K_p N}} \tag{2}$$

In equation 2, $R(s)$ is the set–point, $Y(s)$ is the process output, N is the derivative filter, b and c are weightings that influence the set–point response. This equation may be restructured as

$$U(s) = R(s)F(s) - Y(s)H(s) \tag{3}$$

where $F(s)$ and $H(s)$ are defined by equations 4 and 5 respectively. This representation is illustrated in figure 1, where $G(s)$ is the transfer function representing the plant.

$$F(s) = K_p b + \frac{K_i}{s} + \frac{sK_d}{1 + \frac{sK_d}{K_p N}} c \tag{4}$$

$$H(s) = K_p + \frac{K_i}{s} + \frac{sK_d}{1 + \frac{sK_d}{K_p N}} \tag{5}$$

Clearly, the industrial PID structure corresponds to a 2–DOF control law, where the transfer function $H(s)$ may be chosen to yield optimal regulator performance while the transfer function $F(s)$ can be chosen to yield good servo performance. Note that K_p, K_i, K_d and N appear in both of these transfer functions, hence it is not possible to specify the regulator performance independently from the servo performance. Thus, like the GPC the 2–DOF PID controller is not a true 2–DOF controller. However, the parameters b and c may be utilised to enhance the servo performance and clearly do not affect the regulator performance. For the classical PID controller $F(s)$ and $H(s)$ are equal, given by equation 1.

II(b) Generalised Predictive Controller

The GPC, introduced by Clarke et al. [1], is presented as an alternative to the PID controller. There are three major components in the design of a GPC:

- A model of the system to be controlled. This model is used to predict the system output over the prediction horizon. The GPC uses a *Controlled AutoRegressive and Integrated Moving Average*, CARIMA, model of the form:

  $$A(z^{-1})y(t) = B(z^{-1})u(t-1) + C(z^{-1})\frac{\xi(t)}{\Delta} \tag{6}$$

  where

  B is the plant numerator in z^{-1}

  A is the plant denominator in z^{-1}

  $\xi(t)$ is an uncorrelated random sequence

  Δ differencing operator defined as $1 - z^{-1}$.

  In GPC, if the observer polynomial $T(z^{-1})$ is set equal to $C(z^{-1})$ of equation 6 a minimum variance predictor results. However, since it is difficult to estimate $C(z^{-1})$ accurately, $T(z^{-1})$ is usually chosen for robustness rather than optimality [5, 6] and forms one of the GPC tuning parameters.

- The remaining tuning parameters arise from the definition of the GPC cost function

  $$J = E\left\{\sum_{j=N_1}^{N_2}\left[y(t+j) - w(t+j)\right]^2 + \sum_{j=1}^{N_u}\lambda(j)\left[\Delta u(t+j-1)\right]^2\right\} \tag{7}$$

  where

  N_1 is the minimum cost horizon, subject to $N_1 \leq N_2$

  N_2 is the maximum cost horizon

  N_u is the control horizon, subject to $N_u \leq N_2$

  λ is the control weighting sequence, subject to $\lambda \geq 0$

w future reference trajectory

E is the expectation operator.

- This cost function is then minimised to yield the optimal control output.

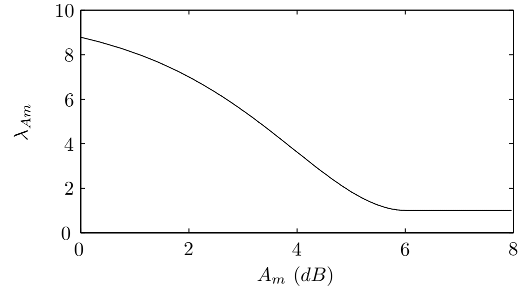
III Controller Design

A variety of techniques exist to design or tune PID controllers. Popular approaches consist of tuning rules [7], Internal Model Control [8], Quantitative Feedback Theory [9] and optimisation procedures [10]. Regardless of the approach taken the design must be robust. Robustness may be achieved through the specification of minimum gain and phase margins [11] or through the optimisation of selected sensitivity functions e.g. $\mathcal{H}_\infty$ control. PID controllers have traditionally been designed using gain and phase margin considerations [11, 12] and therefore this approach is explored in this paper. However, rather than applying tuning rules to achieve the desired gain and phase margin criteria [7], an optimisation based approach will be adopted. This enables the same controller design philosophy to be applied to all three of the controllers under investigation and ensures a fair comparison of controller performance. The optimisation was performed by a GA where the objective function considered is the *Integral of Absolute Error*, IAE, criterion subject to constrains on the gain and phase margins.

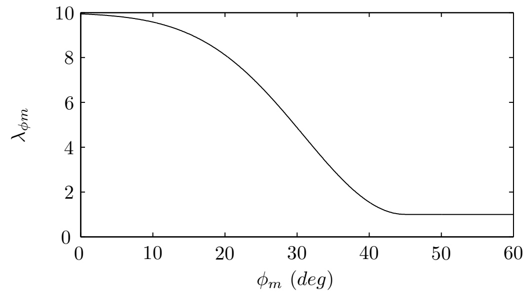
III(a) Genetic Algorithm

The GA approach is an intuitive and mature search and optimisation technique based on the principles of natural evolution and population genetics. Typically the GA starts with little or no knowledge of the correct solution and depends entirely on responses from an interacting environment and its evolution operators to arrive at good solutions. By dealing with several independent points, the GA samples the search space in parallel and hence is less susceptible to converging to a suboptimal solution. In this way the GA has been shown to be capable of locating high performance areas in complex domains without experiencing the difficulties associated with false optima, as may occur with gradient descent techniques. Thus the GA has been recognised as a powerful tool in many control applications.

The structure of the GA was broadly similar to that described by [13] and is popularly known as the simple or canonical GA. The controller parameters were encoded using using Gray coding and an initial, randomly generated population of 100 individuals was used. At each iteration parents were probabilistically selected using stochastic universal sampling, while offsprings were generated using single point crossover (probability = 1) and a single bit mutation (probability=0.0233 for the PID controller, 0.0143 for the 2–DOF, a 0.0130 for the GPC). The algorithm terminated after 300 generations, though in most cases convergence occurred much sooner. With the PID controller the three parameters K_p, K_i and K_d were tuned. Each of these parameters was optimised over the range $0-20$, represented as 10 bit segments with a resolution of 0.01. In the 2–DOF PID controller the parameters K_p, K_i, K_d, b, c had the same resolution as the PID controller parameters. The remaining parameter N was tuned over the range $0-31$ and represented with 5 bits. With the GPC controller the parameters $T(z^{-1})$ and λ had a resolution of 0.01 and N_1, N_2, N_u were specified as integers.



(a) Gain margin



(b) Phase margin

Fig. 2: Constraining factor

III(b) Objective function

The critical component of any optimisation is the design of the objective function to be minimised. As mentioned previously, the objective function employed here incorporates robustness via constrains on the gain and phase margins. These constraints were implemented as follows. First consider the standard Gaussian function

$$y = e^{-x^2} \tag{8}$$

If this function is modified by appropriate scaling (trial and error) it is possible to generate curves such as illustrated in figure 2. Consider figure 2(a). If the gain margin is less than $6(dB)$ then the contribution by this function to the overall cost is large, hence the cost function will be heavily weighted, regardless of the IAE; while if the gain margin is greater than $6(dB)$ the contribution is constant at unity. Similarly, figure 2(b), illustrates a weighting function which directs the search for suitable phase margins. Mathematically these curves are represented by equations 9 and 10

$$\lambda_{Am} = 10 - 9e^{-2\left(\underline{A_m} - A_m\right)^2} \tag{9}$$

$$\lambda_{\phi m} = 10 - 9e^{-0.0025\left(\underline{\phi_m} - \phi_m\right)^2} \tag{10}$$

where A_m is the measured gain margin, $\underline{A_m}$ is the specified minimum value, both expressed as absolute values, ϕ_m is the measured phase margin (in degrees) and $\underline{\phi_m}$ is the specified minimum value.

The IAE of the servo and regulatory response is calculated using equation 11

$$IAE = \sum_{k=0}^{t_1-1} |e(k)| + \sum_{k=t_1}^{t_2} |e(k)| \tag{11}$$

where $e(k)$ is the control error and t_1 is time at which the disturbance, $D(s)$, is applied (see figure 1). Therefore the IAE servo is calculated over the period $0 < t < t_1$, and the IAE regulator is calculated over the period $t_1 \leq t \leq t_2$.

The various objective functions are then

$$J_{PID} = \min_{K_p, K_i, K_d} \{IAE \cdot \lambda_{Am} \cdot \lambda_{\phi m}\} \tag{12}$$

$$J_{2-DOF} = \min_{K_p, K_i, K_d, b, c, N} \{IAE \cdot \lambda_{Am} \cdot \lambda_{\phi m}\} \tag{13}$$

$$J_{GPC} = \min_{N_1, N_2, N_u, \lambda, T(z^{-1})} \{IAE \cdot \lambda_{Am} \cdot \lambda_{\phi m}\} \tag{14}$$

Clearly, if the robustness criteria are satisfied ($\lambda_{Am} = 1$ and $\lambda_{\phi m} = 1$) equations 12–14 reduce to minimising the IAE. The criteria 12–14 may then be interpreted as optimising performance subject to constraints on robustness.

IV Simulation results

The 2–DOF PID and GPC controller structures were evaluated by implementing the controller design equations, 13 and 14 to the list of process models 15–26. These benchmark systems were proposed by Åström and Hägglund [2, 3] and are representative of typical industrial process transfer functions. Since the GPC is a discrete time algorithm these transfer functions had to be sampled. Sampling was performed using the standard Z–transform and the sampling period, h varied from 0.01 sec to 0.5 sec, as specified.

$$G_1(s) = \frac{1}{(s+1)^3} \qquad h = 0.02(sec) \tag{15}$$

$$G_2(s) = \frac{1}{(s+1)^3}e^{-15s} \qquad h = 0.04(sec) \tag{16}$$

$$G_3(s) = \frac{1}{(s+1)(1+0.2s)(1+0.04s)(1+0.008s)} \qquad h = 0.01(sec) \tag{17}$$

$$G_4(s) = \frac{1}{s(s+1)^2} \qquad h = 0.01(sec) \tag{18}$$

$$G_5(s) = \frac{1-2s}{(s+1)^3} \qquad h = 0.1(sec) \tag{19}$$

$$G_6(s) = \frac{9}{(s+1)(s^2+2s+9)} \qquad h = 0.1(sec) \tag{20}$$

$$G_7(s) = e^{-s} \qquad h = 0.1(sec) \tag{21}$$

$$G_8(s) = \frac{e^{-s}}{s} \qquad h = 0.02(sec) \tag{22}$$

$$G_9(s) = \frac{1}{10s+1} \qquad h = 0.5(sec) \tag{23}$$

$$G_{10}(s) = \frac{100}{(s+10)^2}\left(\frac{1}{s+1} + \frac{0.5}{s+0.05}\right) \qquad h = 0.01(sec) \tag{24}$$

$$G_{11}(s) = \frac{150}{(s+10)^2(s+1)} \qquad h = 0.01(sec) \tag{25}$$

$$G_{12}(s) = \frac{(s+6)^2}{s(s+1)^2(s+36)} \qquad h = 0.01(sec) \tag{26}$$

Prior to discussing the results it is appropriate to make a few brief comments regarding implementation details. The cost function of section III(b) was optimised in the MATLAB/Simulink environment using the GA. Controller evaluation was also performed in this environment with the gain and phase margins measured in MATLAB and the IAE calculated in Simulink. Both the PID and 2–DOF controllers were implemented in continuous–time while the GPC controller was implemented as a discrete time system.

For models with time delay, G_2, G_7, G_8, an 8^{th} order Padé approximation was used in the calculation of the gain and phase margins. The control signal was constrained to the range $-10 \leq u(k) \leq 10$. Robustness was achieved by considering requirements for the gain and phase margin. For example, [14] specifies that typical minimum requirements are a gain margin $\underline{A_m} = 1.7$ $(4.6dB)$ and a phase margin $\underline{\phi_m} = 30°$. In the papers [11, 12] the authors assumed $A_m = 3$ $(9.5dB)$ and $\phi_m = 60°$ in their controller design, while [4, p. 126] recommended that the gain margin should be in the range $A_m = 2-5$ $(6dB - 14dB)$ and that the phase margin should be $\phi_m = 30° - 60°$. Given the variation in these recommendations two designs were considered. The first design specified a minimum gain margin of $\underline{A_m} = 2$ $(6dB)$ and a minimum phase margin $\underline{\phi_m} = 45°$. In the second design, the phase

System	PID			2–DOF PID			GPC		
	IAE	A_m	ϕ_m	IAE	A_m	ϕ_m	IAE	A_m	ϕ_m
$G_1(s)$	2.08	∞	44	1.62	14.64	45	1.54	6.83	44
$G_2(s)$	42.9	6.27	64	41	6.02	64	33.52	6.00	59
$G_3(s)$	0.409	22.70	45	0.27	15.59	45	0.208	11.30	45
$G_4(s)$	5.46	∞	44	3.51	14.54	45	0.838	11.87	71
$G_5(s)$	10.7	5.94	58	9.94	5.88	55	8.79	6.00	59
$G_6(s)$	1.44	∞	45	1.15	13.34	45	0.859	6.97	45
$G_7(s)$	2.59	∞	63	2.55	7.05	63	2.11	6.02	60
$G_8(s)$	7.39	8.02	47	4.74	5.82	44	2.83	9.33	71
$G_9(s)$	1.03	∞	78	0.533	∞	69	0.60	6.02	45
$G_{10}(s)$	0.38	∞	45	0.3	13.04	45	0.195	12.65	51
$G_{11}(s)$	0.351	∞	45	0.259	25.57	44	0.166	9.26	45
$G_{12}(s)$	1.09	∞	45	0.729	∞	45	0.531	7.69	52

Table 1: The optimised IAE with $\underline{A_m} = 6dB$ and $\underline{\phi_m} = 45°$

System	PID			2–DOF PID			GPC		
	IAE	A_m	ϕ_m	IAE	A_m	ϕ_m	IAE	A_m	ϕ_m
$G_1(s)$	2.05	∞	45	1.63	14.59	45	1.59	14.02	45
$G_2(s)$	53.9	14.00	63	53.7	14.01	63	55.3	14.00	75
$G_3(s)$	0.407	22.00	45	0.256	17.02	45	0.233	14.00	53
$G_4(s)$	4.47	∞	45	3.51	14.76	45	0.838	14.00	70
$G_5(s)$	15.6	14.01	60	15.6	14.02	57	12.8	14.17	67
$G_6(s)$	1.47	∞	45	1.15	14.22	45	0.921	14.08	45
$G_7(s)$	2.56	∞	62	3.09	14.19	61	2.51	14.44	62
$G_8(s)$	7.39	8.02	47	8.93	13.99	44	2.5	14.81	∞
$G_9(s)$	1.03	∞	77	0.498	∞	81	0.68	13.99	48
$G_{10}(s)$	0.377	∞	45	0.331	13.99	45	0.195	14.01	50
$G_{11}(s)$	0.349	∞	45	0.297	14.33	45	0.189	14.01	51
$G_{12}(s)$	1.09	∞	45	0.588	∞	48	0.531	14.06	53

Table 2: The optimised IAE with $\underline{A_m} = 14dB$ and $\underline{\phi_m} = 45°$

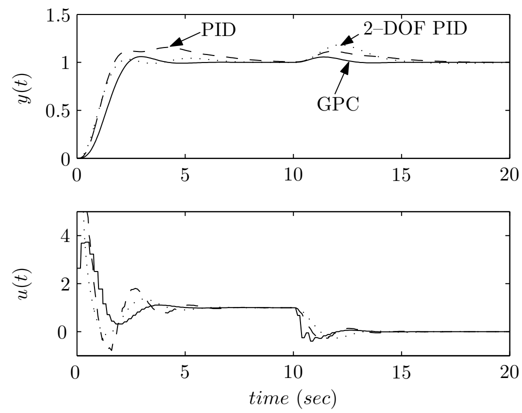


Fig. 3: Set–point following for $G_1(s)$, $\underline{A_m} = 6dB$

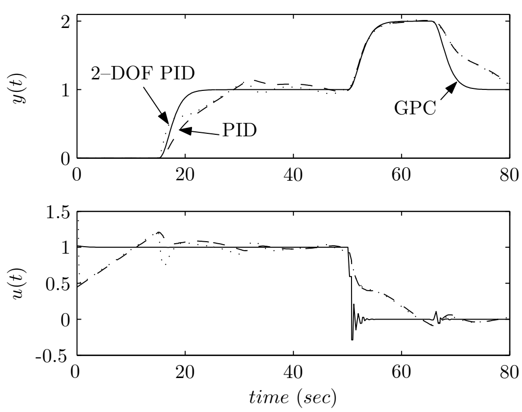


Fig. 4: Set–point following for $G_2(s)$, $\underline{A_m} = 6dB$

margin criteria remained unchanged, but the gain margin was increased to $\underline{A_m} = 5$ $(14dB)$.

The simulation results for the first design ($\underline{A_m} = 6dB$, $\underline{\phi_m} = 45°$) are presented in table 1. Clearly the design was successful in that the specified minimum gain and phase margins were achieved for all three controllers. If the integral of absolute error criteria are compared, it is evident that the GPC controller outperforms the 2–DOF PID controller which in turn outperforms the 1–DOF PID controller. It is not surprising that the 2–DOF controllers should outperform the 1–DOF controller, however the GPC controller performs significantly better than the 2–DOF PID controller — on average the IAE was reduced by 22%. The most significant reductions occurred for the two integrating plants, $G_4(s)$ where the IAE was reduced by 76% while for $G_8(s)$ a reduction of approximately 40% was achieved. A cursory examination of the loop gain margins reveals that on average the A_m associated with the 2–DOF PID controller is larger than the corresponding value of the GPC controller. This would suggest that a classic compromise — robustness versus performance — exists. However, this is unlikely to be the case as the phase margin associated with the GPC design, is on average, the same or greater than that for the 2–DOF PID controller.

To investigate the robustness versus performance idea further a second design was performed where the minimum gain margin was increased to $14dB$. The results are summarised in table 2. Surprisingly, the results in this case are even clearer. Ignoring the 1–DOF PID controller for the present, it is evident that the remaining two controllers have very similar gain margins, the phase margin is, in general, larger for the GPC controller and the GPC IAE is reduced, on average, by approximately 15%.

A selection of closed–loop responses are illustrated in figures 3, 4 and 5 where the solid line represents the response of the GPC controller, the dashed line that of the 1–DOF PID controller and the dotted line the 2–DOF PID controller. In each of these figures the upper plot illustrates the closed–loop response to a unit step input occurring at $t = 0$ and to a unit step disturbance. In figures 3 and 5 the disturbance was applied at $t = 10(sec)$ while in figure 4 the disturbance occurs at time $t = 35(sec)$. Also shown are the corresponding control signals. The superior performance of the GPC controller is clearly evident in each case.

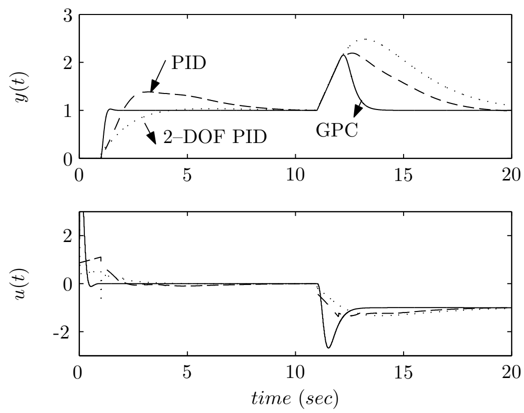


Fig. 5: Set–point following for $G_8(s)$, $\underline{A_m} = 14dB$

V Conclusions

From figures 3, 4 and 5 and the results listed in tables 1 and 2 it is evident that the design proposed in this paper is very effective. Despite the wide range of process dynamics: stable, inverse unstable, integrating, long time delay, stable and consistent closed–loop performance was obtained for all of the systems examined. This is particulary true for both of the 2–DOF controllers.

Tables 1 and 2 clearly indicate that the GPC controller outperforms the other two controllers. This enhanced performance relative to the simple 1–DOF PID controller is not surprising and has been well established elsewhere. The GPC and 2–DOF PID algorithm both have similar structures (they both have two degrees of freedom) and a comparable number of tuning parameters and in this context the superior performance of the GPC was a little surprising and would suggest that this controller may offer some real benefit to the process industry.

This work, while providing some insight into the relative merits of the controllers is limited by the fact that perfect modelling was assumed. It is anticipated that if model uncertainty was incorporated and robust performance considered the results might vary. The next stage of this work will indirectly address this issue by evaluating the proposed design and the three controllers on a number of real–time systems, a level control problem, a flexible link and a pilot scale pasteurisation unit.

References

[1] D. W. Clarke, C. Mohtadi, and P. Tuffs, "Generalized predictive control — Part I and II," *Automatica*, vol. 23, no. 2, pp. 137–160, 1987.

[2] K. J. Åström, H. Panagopoulos, and T. Hägglund, "Design of PI controllers based on non–convex optimization," *Automatica*, vol. 34, no. 5, pp. 585–601, 1998.

[3] K. J. Åström and T. Hägglund, "Benchmark systems for PID control," in *IFAC Workshop on Digital Control: Past Present and Future of PID Control*, (Terrassa, Spain), pp. 165–166, 2000.

[4] K. J. Åström and T. Hägglund, *PID controllers: theory, design, and tuning*. Instrument Society of America, 2^{nd} ed., 1995.

[5] B. Robinson and D. W. Clarke, "Robustness effects of a prefilter in generalized predictive control," *IEE Proceedings D*, vol. 138, pp. 2–8, January 1991.

[6] T.-W. Yoon and D. W. Clarke, "Observer design in receding — horizon predictive control," *International Journal of Control*, vol. 61, no. 1, pp. 171–191, 1995.

[7] A. O'Dwyer, *Handbook of PI and PID Controller Tuning Rules*. ISBN 1-86094-342-X, Imperial College Press, 2003.

[8] D. Rivera, M. Morari, and S. Skogestad, "Internal model control, PID controller design," *Industrial and Engineering Chemistry Process Design and Development*, vol. 25, no. 1, pp. 252–265, 1986.

[9] Q.-G. Wang, C. Hang, Y. Yang, and J. He, "Quantitative robust stability analysis and PID controller design," *IEE Proceedings, Control Theory & Applications*, vol. 149, pp. 3–7, January 2002.

[10] G. P. Liu and S. Daley, "Optimal–tuning PID controller design in the frequency domain with application to a rotary hydraulic system," *Control Engineering Practice*, vol. 7, pp. 817–926, July 1999.

[11] W. K. Ho, C. C. Hang, and L. S. Cao, "Tuning of PID controllers based on gain and phase margin specifications," *Automatica*, vol. 31, no. 3, pp. 497–502, 1995.

[12] Q.-G. Wang, H.-W. Fung, and Y. Zhang, "PID tuning with exact gain and phase margin," *ISA Transactions*, vol. 38, pp. 243–249, 1999.

[13] A. Chipperfield, P. Fleming, H. Pohlheim, and C. Fonseca, *Genetic Algorithm Toolbox user's guide*. Department of Automatic Control and Systems Engineering, University of Sheffield, 1.2 ed., 1994.

[14] D. Seborg, T. Edgar, and D. Mellichamp, *Process Dynamics and Control*. New York: John Wiley & Sons, 1989.